

## Receptacles\*

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**1. The Question.** In (Cartwright, 1975), Richard Cartwright invites us to understand by a *receptacle* a region of space possibly exactly occupied by a material object.<sup>1</sup> I would like to discuss the question of which regions of space are receptacles. At first glance, the answer to our question would appear to be constrained by what Cartwright seemed to regard as platitudes:

Plainly, not every region is a receptacle. The null region is not; neither is any region that consists of a single point or, for that matter, of any finite number of points. Nor are higher cardinalities by themselves sufficient: no region exceeds a straight line in sheer number of members; yet straight lines, along with curves and surfaces, are not receptacles. How then are receptacles to be characterized?<sup>2</sup>

But the characterization of receptacles would be trivial, if contrary to what is suggested above, *all* regions of space are receptacles. This is what has recently come to be known as the liberal view of receptacles:

LIBERAL VIEW OF RECEPTACLES: All regions of space are receptacles.

On the liberal view, pace Cartwright, single points, finite sets of points and straight lines, along with curves and surfaces are in fact receptacles. The liberal view has recently been defended, for example, in (Hudson, 2002) and (Hudson, 2006) on the grounds that it is forced upon us by independently plausible principles concerning the interaction of mereological relations on material objects with location. An independent argument for the thesis

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<sup>1</sup>Presumably spatial location should be relativized to a time. In order to avoid cumbersome terminology, I will leave the temporal relativizations of spatial location and the part-whole relation implicit.

<sup>2</sup>(Cartwright, 1975), p. 171. Page references are to the reprinted version.

that the liberal view is, in fact, entailed by such principles has been recently offered in (Bays, 2003).

In this paper, I would like to investigate the extent to which the answer to our question is constrained by various mereological relations on material objects and their interaction with the relation of exact location. I will look, in particular, into the prospects of moderate views of receptacles in line with Cartwright's apparent platitudes: that neither single points, nor lines or surfaces are receptacles.

Here is the plan for the paper. First, I will identify three candidate moderate views of receptacles, and I will articulate a serious challenge for them posed by the combination of classical extensional mereology with apparently reasonable constraints on the interaction of various mereological relations on material objects with the relation of exact location.<sup>3</sup> Then I will indicate how to meet the challenge on behalf of two of the three candidate moderate views of receptacles. This will involve the rejection of what would have otherwise seemed a very attractive thought on the interaction of the relation a mereological fusion of material objects bears to them and the relation of exact location. I will argue that, on closer inspection, the principle derives its apparent plausibility from a closely related but crucially weaker principle. Moreover, when taken seriously, its necessitation threatens the very possibility of atomless gunk, by which I mean, as usual, material objects all of whose parts have further proper parts. Finally, I will conclude that two moderate views of receptacles are live answers to Cartwright's question as far as mereological relations on material objects and their interaction with exact location are concerned.

**2. Three Moderate Views of Receptacles.** The first task is to identify moderately plausible moderate views of receptacles on which neither single points, nor lines or surfaces qualify as receptacles. Richard Cartwright, inspired by Alfred Tarski's interpretation of the geometry of solids in (Tarski, 1929), developed his own conjecture: a region of space is a receptacle if and only if it is an *open domain*. That will be our departure point.

In the meantime, we will need some definitions to be able to understand Cartwright's

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<sup>3</sup>Some constraints have been formulated in (Hudson, 2002), (Hudson, 2006) and (Bays, 2003), while others are only tacitly used in their arguments.

conjecture and to appreciate the challenges to which it is subject.

First we need a distinction between open and closed regions of space:

An *open sphere about  $p$*  is a region the members of which are all and only those points that are less than some fixed distance from  $p$ .

A point  $p$  is a *boundary point* of a region  $R$  if and only if every sphere about  $p$  contains points from  $R$  and the complement of  $R$ .

A region  $R$  is *open* if and only if it contains none of its boundary points.

A region  $R$  is *closed* if and only if it contains all of its boundary points.

This distinction suggests two tentative answers to our question:

OPEN VIEW: A region of space is a receptacle if and only if it is open

CLOSED VIEW: A region of space is a receptacle if and only if it is closed

Neither proposal survives closer inspection. The closed view is not a live candidate for our purposes because it would make a single point region a receptacle since it is closed. The open view seems to fare better in this respect because a region of space is open only if it is extended in all spatial dimensions there are. Unfortunately, the open view is, by Cartwright's own lights, still unacceptable. One problem with it is that it would allow, for example, a sphere minus its center point to count as a receptacle. To exclude such regions as receptacles, Cartwright recommended the addition of a further constraint. To that purpose, we need yet a few more definitions:

The *interior* of  $R$  is the set of points of  $R$  that are not boundary points of  $R$ .

The *closure* of  $R$  is the union of  $R$  with the set of all its boundary points.

An *open domain* is an open region that is identical with the interior of its closure.

We are finally in a position to state Cartwright's final conjecture in (Cartwright, 1975):

OPEN DOMAIN VIEW: A region of space is a receptacle if and only if it is a non-null open domain.<sup>4</sup>

This is the first moderate view of receptacles I would like to call attention to.

Though more plausible than the open view, Cartwright's open domain view is not beyond question. We will briefly consider two different objections to it. One objection is that the open domain view is mistaken since *all* receptacles are in fact closed. The other, more common objection is that the open domain view is more conservative than necessary: *some* but not all receptacles are in fact closed.

The first objection suggests another moderate alternative to the open domain view:

CLOSED DOMAIN VIEW: A region of space is a receptacle if and only if it is the closure of a non-null open domain.<sup>5</sup>

The closed domain view is immune to the main problem that afflicted the closed view of receptacles. While all closed domains are closed, no zero- or one- or two-dimensional closed region qualifies as a closed domain because none of them is the closure of a non-null open domain. At all events, the closed domain view is our second moderate view of receptacles.

One might wonder whether there are principled reasons to prefer Cartwright's conjecture over the closed domain view. After making the observation that Descartes apparently thought of material objects as open, Cartwright tells us that he will follow Descartes without argument.<sup>6</sup> And he added in reference to the question of whether all receptacles are open or closed that "the issue seems hardly worthy of serious dispute."<sup>7</sup>

How could the question of whether receptacles are open or closed be "hardly worthy of serious dispute"? This might be the case, I think, if the distinction between open and

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<sup>4</sup>Open domains are sometimes referred to as "open regular sets." Since the null set lacks boundary points, it is both open and closed. It is open because it contains none of its boundary points; closed because it contains all of them. Moreover, since the null set is identical to the interior of its closure, it is an open domain as well.

<sup>5</sup>We need to require the open domain in question to be non-null to make sure closed domains are extended in all spatial dimensions.

<sup>6</sup>Cartwright cites a passage from the *Principles of Philosophy* where after explaining that the "external place" of a body is "the superficies of the surrounding body", Descartes remarks that "by superficies we do not here mean any portion of the surrounding body, but merely the extremity which is between the surrounding body and that surrounded." (*Principles of Philosophy*, Part 2, Principle 15.)

<sup>7</sup>(Cartwright, 1975).

closed regions turned out to be a distinction without a difference. We have been implicitly making a familiar assumption according to which space consists of extensionless points and sets thereof that exhibit the structure of ordinary three-dimensional Euclidean space. The success and power of this model of space is unquestioned. But perhaps nothing in space corresponds to extensionless points and points should instead be construed as sets of regions. Examples of this alternative approach to space abound in the literature.<sup>8</sup> If space is pointless and all regions are extended, then the distinction between open and closed regions would collapse and would no longer correspond to a difference in reality. And this would render moot the question of whether receptacles should be open or closed.

But if the distinction between open and closed regions is not merely an artifact of our representation of space but rather captures a distinction in reality, then one may object to Cartwright's open domain view on the grounds that the restriction to open regions is uncalled for. This is the second objection to the open domain view. Since not all closed regions are objectionable as candidate receptacles in the way points, lines and surfaces are, there is no reason to exclude all closed regions in the first place. Why, to use a phrase Cartwright uses in a similar context, impose bounds on the possibilities?

It is not difficult to improve on the two moderate views of receptacles listed above by allowing both open and closed regions as receptacles while nevertheless excluding all of the regions Cartwright identified as problematic:

DOMAIN VIEW: A region of space is a receptacle if and only if its non-null interior is included in its closure.<sup>9</sup>

Unlike the views mentioned above, the domain view makes provision for open, closed and even partially open receptacles. And yet, the proposal manages to exclude points, lines and

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<sup>8</sup>For two prominent examples, consider (Whitehead, 1929) and (Tarski, 1929), on which more later.

<sup>9</sup>Some partially open receptacles might be a source of reservation. For the view now under consideration seems to allow for partially open receptacles with a single, finitely many and even countably many boundary points. And such regions may strike one as objectionable in much the way some open regions without inner boundary points seem to be. Compare this with a more liberal view briefly noted, for example, in (Hawthorne and Weatherston, 2004) on which a region is a receptacle if and only if its interior is a non-null open domain. One problem with this view is that it would allow the union of a closed domain with an arbitrary set of boundary points to count as a receptacle. For example, the union of a closed spherical domain with a line of infinite length would count as a receptacle on the more liberal view but not on the domain view stated above. I am indebted to Ignacio Jané for this observation.

planes as receptacles on the grounds that they lack a non-null interior—all of their points are boundary points. Moreover, when we consider the case, for example, of a sphere minus its center point, we would be in a position to rule it out as a receptacle on the grounds that its interior is different from the interior of its closure, which, unlike the former, contains the center point in addition to all of the other points in the sphere. What we have called the domain view is our third moderate view of receptacles.

**3. The Challenge.** I would like to look at the question of whether classical extensional mereology, in combination with fundamental principles of location with a claim to govern the interaction of various mereological relations on material objects with exact location, is compatible with a moderate view of receptacles on which neither single points nor lines nor surfaces count as receptacles. And, in particular, I would like to mount what I take to be the best case for the claim that, in the presence of fundamental principles of location, classical extensional mereology forces upon us the liberal view of receptacles.

To that purpose, I will make use of two principles of classical extensional mereology:

- (1) WEAK SUPPLEMENTATION: If  $x$  is a proper part of  $y$ , then there is some proper part of  $y$  that is discrete from  $x$ .
- (2) UNRESTRICTED COMPOSITION: Whenever there are some objects, there is a mereological fusion of them.

And I will combine them with three common presuppositions with respect to the interaction of various mereological relations on material objects and location:

- (3) PARTS:  $x$  is part of  $y$  if and only if the region of space exactly occupied by  $x$  is included in the region of space exactly occupied by  $y$ .
- (4) OVERLAP:  $x$  and  $y$  overlap if and only if the regions of space they exactly occupy have a non-null intersection.<sup>10</sup>

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<sup>10</sup>(4) need not follow from (3), or at least not on the closed domain view: If  $x$  and  $y$  are two closed continuous cubes with an adjacent side and no two-dimensional regions are occupiable, then they will *not* overlap despite the fact that the regions of space they exactly occupy do. More on this on section 5.

- (5) FUSIONS: Let  $X$  be the mereological fusion of a non-null family  $\mathcal{F}$  of material objects. Then  $X$  exactly occupies the set-theoretic union of the regions of space exactly occupied by the material objects in the family  $\mathcal{F}$ .<sup>11</sup>

When we identify the part-whole relation on regions of space of ordinary three-dimensional Euclidean space with the subset relation on sets of extensionless points, then we are in a position to motivate (3), (4) and (5) as members of a single hypothesis to the effect that three mereological relations on material objects, part-whole, overlap and fusion, are perfectly aligned with their counterparts on regions of space: subset, set-theoretic intersection and set-theoretic union. This is because the characterization of the part-whole relation on regions in terms of the subset relation forces upon us the identification of overlap with the relation of set-theoretic intersection and fusion with set-theoretic union.

Before I raise what I take to be the central challenge for moderate views of receptacles generally, notice that (1), (3) and (4) are in direct tension with the domain view. The problem is not difficult to state. Take a closed domain and consider its open interior. Both the closed domain and its interior are receptacles on the domain view. Suppose further two such regions are simultaneously exactly occupied by distinct material objects.<sup>12</sup> By (3) the material object exactly occupying the interior of the closed domain is a proper part of the material object exactly occupying the closed domain. But (1) tells us that some proper part of the latter object exists that is discrete from the former. Unfortunately, by (4), such a material object would have to exactly occupy some subregion of the two-dimensional surface of the closed domain.

But the challenge I would like to discuss now is different and concerns moderate views generally. The challenge arises from the observation that (1) – (5) jointly entail:

- (★) If some material object  $A$  occupies a point  $p$ , some mereologically atomic — or simple — part of  $A$  occupies  $p$  as well.<sup>13</sup>

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<sup>11</sup>Talk of a family of material objects could be paraphrased with the help of plurals. But no harm would be done for present purposes if one took such talk to involve reference to sets of material objects.

<sup>12</sup>Admittedly, the possibility that the two regions be simultaneously exactly occupied by distinct material objects is not guaranteed merely by the fact that the two regions are receptacles. However, it is difficult to think of reasons that would help us exclude this further possibility as a matter of principle.

<sup>13</sup>I am grateful to Ignacio Jané for discussion.

The argument is this:

Let  $A$  be a material object that occupies some point  $p$ . We will argue that some atomic part of  $A$  occupies  $p$ . If  $A$  is a mereological atom, we are done. Therefore, let us assume that  $A$  has some proper part  $B$ . Now:

(i) Not all proper parts of  $A$  occupy  $p$ .

Since  $B$  is a proper part of  $A$ , by (3),  $B$  occupies some proper subregion of the region exactly occupied by  $A$ . If  $B$  does not occupy  $p$ , then we are done. Otherwise, by (1), there is some proper part of  $A$ ,  $C$ , which is discrete from  $B$ . Since, by (4),  $B$  and  $C$  occupy disjoint regions and  $B$  occupies  $p$ ,  $C$  is a proper part of  $A$  that does not occupy  $p$ .

Let  $X$  be the mereological fusion of all and only those parts of  $A$  which do not occupy  $p$ , which, in the presence of (i), is guaranteed to exist by (2). Then:

(ii)  $X$  is a proper part of  $A$  which does not occupy  $p$ .

$X$  is part of  $A$  because it is a mereological fusion of parts of  $A$ . It is a proper part because, by (5),  $X$ , unlike  $A$ , does not occupy  $p$ .

There is, by (1), some proper part of  $A$ ,  $S$ , which is discrete from  $X$ . But then:

(iii)  $S$  is an atomic part of  $A$  that occupies  $p$ .

- $S$  occupies  $p$ . Otherwise, since  $S$  is discrete from  $X$ ,  $X$  would not be the mereological fusion of *all* parts of  $A$  that do not occupy  $p$ .
- $S$  is atomic. For otherwise,  $S$  would have some proper part  $T$ . And if  $T$  does not occupy  $p$ , then, since  $T$  is discrete from  $X$ ,  $X$  would not be the fusion of *all* parts of  $A$  that do not occupy  $p$ . But if  $T$  occupies  $p$ , then, by (1),  $S$  has some other proper part  $V$  that is discrete from  $T$ . But since, by (4),  $V$  does not occupy  $p$ , we have that  $V$ , which is part of  $S$ , is a proper part of  $A$  that does not occupy  $p$  and is discrete from  $X$ . But this again contradicts the assumption that  $X$  is the mereological fusion of *all* parts of  $A$  that do not occupy  $p$ .

This result is remarkable for at least two reasons.<sup>14</sup>

In the first place, when we combine (1) – (5) with a further premise on the interaction of the part-whole relation with location, we seem to have an argument for non-extended simples. The further assumption is this:

(6) NO EXTENDED SIMPLES: There are no extended simples.

When we understand “extended simple” to mean a simple exactly located in a region of space that contains more than one point, then we have an argument for the existence of single-point simples, and, a fortiori, an argument for single-point receptacles.<sup>15</sup> And once we agree that regions that consist of a single point are receptacles, the door is open for regions that consist of some finite number of points to qualify as receptacles along with lines, curves and surfaces.

However, on a more restrictive interpretation, “extended simple” could be taken to mean a simple exactly located in a region of space of zero topological dimension, in which case we would have an argument for a more modest thesis according to which some material objects, and, a fortiori, some receptacles have zero dimension. But even the more restrictive interpretation would clearly be in tension with Cartwright’s apparent platitudes.

In the second place, and perhaps more remarkably, (1) – (5) jointly entail atomism:

(7) ATOMISM: No material objects exist all of whose parts have further proper parts.

The argument is straightforward:

Assume for a reductio that  $A$  is a material object all of whose parts have further proper parts. Since all material objects occupy some region of space, let  $p$  be a point  $A$  occupies. By (3) – (5), some atomic part of  $A$  occupies  $p$ . But, unfortunately, this atomic part of  $A$  lacks further proper parts. Contradiction.

This, I will suggest, is quite significant in view of the fact that (1) – (3) might perhaps be thought to state a priori constraints on the interaction of the part-whole relation and

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<sup>14</sup>This, it seems to me, is what lies at the heart of Bays’s result in (Bays, 2003). The advantage of the present argument is that it avoids the need for transfinite recursion and the subsequent appeal to the axiom of choice. I have postponed a discussion of Bays’s argument to an appendix.

<sup>15</sup>This, I think, is how “extended simple” is understood in (Hudson, 2002) and (Hudson, 2006).

location. On this view, each of (1) – (3) would presumably qualify as necessary truths. But we cannot confer the same status to (4) and (5) unless we are prepared to admit the necessary truth of atomism. Since (1) – (5) jointly entail atomism, atomism will be a necessary truth if all of (1) – (5) are. And yet, one would have thought that no a priori constraints on the interaction of the part-whole relation with location should exclude the very possibility of atomless gunk. This, I will suggest, provides us with some reason to question (4) and (5).<sup>16</sup>

**4. Moderate Views and Classical Extensional Mereology.** The challenge of the last section raises the question whether, in the presence of (3) and (6), moderate views of receptacles are even compatible with the principles of classical extensional mereology. The purpose of this section is to answer this question in the affirmative.

For present purposes, I will assume a succinct formulation of classical extensional mereology due to Alfred Tarski in (Tarski, 1929) and later reformulated in (Lewis, 1991) with the help of plural quantification:

TRANSITIVITY: If  $x$  is part of  $y$  and  $y$  is part of  $z$ , then  $x$  is part of  $z$ ,

UNRESTRICTED COMPOSITION: Whenever there are some objects, there is a mereological fusion of them,

UNIQUENESS OF COMPOSITION: It never happens that the same objects have two different mereological fusions,

where “overlap” is taken as a primitive and “part” and “fusion” are defined thus:

DEFINITION:  $x$  is *part* of  $y$  iff everything that overlaps  $x$  overlaps  $y$ .

DEFINITION:  $x$  is a mereological *fusion* of some objects iff  $x$  has all of them as parts and has no part that is disjoint from each of them.

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<sup>16</sup>It seems to be that (3) rests on firmer ground than (4) and (5) because it gives expression to a more fundamental view of the part-whole relation on material objects as a spatial relation. As we will see in the next sections, this view of the part-whole relation is consistent with other alternatives to (4) and (5).

Weak supplementation is a consequence of uniqueness of composition and definitions.<sup>17</sup>

Tarski provided this axiomatization of classical extensional mereology against the background of what he called the *geometry of solids*, which consisted of four postulates in addition to the axioms of classical extensional mereology.<sup>18</sup> Tarski's geometrical system dispensed with points, lines and surfaces as fundamental constituents of space in favor of spheres. He defined solids as arbitrary mereological fusions of spheres, which he conceived of as intuitive correlates of open domains of ordinary three-dimensional Euclidean geometry.

But Tarski's key observation for present purposes is just that the geometry of solids has a model in ordinary three-dimensional Euclidean space when "solid" is interpreted to refer to non-null open domains and "part" is taken to stand for the subset relation. This is important for our purposes because again Tarski's geometry of solids included all of the axioms of classical extensional mereology. It follows that the family of non-null open domains of three-dimensional Euclidean geometry constitutes a model of classical extensional mereology when we take "part" to stand for the subset relation among open domains.

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<sup>17</sup>*Proof sketch.* Suppose  $a$  is a proper part of  $b$ . Assume by reductio that every proper part of  $b$  overlaps  $a$ . Then  $b$  will be a fusion of  $a$ . Since  $a$  is a proper part of  $b$ ,  $a$  is a part of  $b$ . Moreover, if  $z$  is a part of  $b$ , then either  $z = b$  or  $z$  is a proper part of  $b$ . In either case, given our assumptions,  $z$  overlaps  $a$ . Thus  $a$  is part of  $b$  and every part of  $b$  overlaps  $a$ . But of course  $a$  is itself a fusion of  $a$  since  $a$  has  $a$  as a part and every part of  $a$  overlaps  $a$ . By uniqueness of composition,  $a = b$  and hence  $a$  is not a proper part of  $b$  after all.

<sup>18</sup>Tarski defined "point" and "interior point" thus:

DEFINITION: A point is the class of all spheres which are concentric with a given sphere.

DEFINITION: The point  $p$  is an interior point of the solid  $B$  if there exists a sphere  $A$  which is at the same time an element of the point  $p$  and a part of the solid  $B$ .

Tarski's four postulates are:

POSTULATE 1: The notions of point and of equidistance of two points from a third satisfy all the postulates of ordinary Euclidean geometry of three dimensions.

POSTULATE 2: If  $A$  is a solid, the class  $\alpha$  of all interior points of  $A$  is a non-null regular open set. (A *regular* open set is an open domain, i.e., a set that coincides with the interior of its closure.)

POSTULATE 3: If the class of points  $\alpha$  is a non-null regular open set, there exists a solid  $A$  such that  $\alpha$  is the class of all its interior points.

POSTULATE 4: If  $A$  and  $B$  are solids, and all the interior points of  $A$  are at the same time interior points of  $B$ , then  $A$  is part of  $B$ .

This observation alone guarantees that the family of non-null open domains under the subset relation satisfies each of (1), (2), (3) and (4). Moreover, since *all* open domains have further open domains as subsets, we have that Tarski's model satisfies (6) as well. As a result, contrary to what may have been suggested in the literature, we have that (1), (2), (3), (4) and (6) do not by themselves entail the existence of single point receptacles or the liberal view of receptacles.<sup>19</sup>

Does Tarski's observation call into question the argument from last section? And, in particular, does it challenge the claim that (1) - (5) entails atomism? Or the claim that (1) - (6) entails the existence of unextended simples? No. For one reason: The family of non-null open domains under the subset relation is not a model of (5). In order to appreciate this, it may be helpful to state the formal result that lies at the heart of Tarski's observation:

OPEN DOMAIN MEREOLGY:

Let  $\mathcal{M}$  be a non-null family of non-null open domains of a topological space. And suppose  $\mathcal{M}$  is such that (i) the interior of the closure of the union of each non-null family of elements of  $\mathcal{M}$  is itself in  $\mathcal{M}$ ; (ii) if  $A$  and  $B$  are members of  $\mathcal{M}$  such that  $A - B$  is non-empty, then the interior of the closure of  $A - B$  is in  $\mathcal{M}$ . Then  $(\mathcal{M}, \subseteq)$  is a model of classical extensional mereology with respect to which the operation of mereological fusion is interpreted by the function that assigns to each non-null family of non-null open domains in  $\mathcal{M}$  the interior of the closure of its union.<sup>20</sup>

Tarski's observation is a special instance of this result. For if  $\mathcal{O}$  is the family of *all* non-null open domains of ordinary three-dimensional Euclidean space, then  $\mathcal{O}$  is closed under operations (i) and (ii). Therefore:  $(\mathcal{O}, \subseteq)$  is a model of (1), (2), (3), (4) and (6): because  $\subseteq$  is transitive and extensional, it is not difficult to verify that the model immediately

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<sup>19</sup>Arguments for this claim occur in (Hudson, 2002), (Hudson, 2006) and (Bays, 2003). I will discuss some of the arguments in (Hudson, 2002) and (Hudson, 2006) in the next section. As for the argument in (Bays, 2003), suffice it to say for now that it makes tacit but nonetheless crucial use of both (4) and (5).

<sup>20</sup>It follows that the operation of mereological complement is interpreted by the function that assigns to each open domain the interior of the closure of its set-theoretic complement. This is a generalization of Tarski's THEOREM B in p. 29 of (Tarski, 1929).

satisfies Transitivity and Uniqueness of Composition. (i) and (ii), however, makes sure that Unrestricted Composition is satisfied as well. Moreover, it is a model of (4) since two open domains have a non-null intersection if and only if they include a common open domain in which case they overlap. Finally, the model satisfies (6) just because all open domains have further open domains as proper subsets.

However, the family of all non-null open domains under the subset relation does not constitute a model of (5). At most, it provides us with a model of:

- (8) Let  $X$  be the mereological fusion of a non-null family  $\mathcal{F}$  of material objects. Then  $X$  exactly occupies the interior of the closure of the set-theoretic union of the regions of space exactly occupied by the material objects in the family  $\mathcal{F}$ .

To appreciate the import of this principle, it will be helpful to look at a specific example. Let Sphere be an open spherical region which is identical to the interior of its closure. And let Easterner and Westerner, respectively, be two open hemispheres of Sphere separated by a two-dimensional region. Each of the three regions are open domains. Assume further that EASTERNER and WESTERNER are, respectively, two material objects which exactly occupy Easterner and Westerner. What is the region of space exactly occupied by the mereological fusion of EASTERNER and WESTERNER?

We have two competing answers. According to (5), the mereological fusion of EASTERNER and WESTERNER should exactly occupy the set-theoretic union of Easterner and Westerner, which is all of Sphere but a two-dimensional region. Therefore, the fusion exactly occupies a proper part of Sphere which is itself not an open domain. However, according to (8), the mereological fusion of EASTERNER and WESTERNER should exactly occupy the interior of the closure of the set-theoretic union of Easterner and Westerner, which is all of Sphere.<sup>21</sup> Thus, if SPHERE is a material object which exactly occupies Sphere, the mereological fusion of EASTERNER and WESTERNER is no other than SPHERE.

But this is exactly how it should be on the open domain view. For Sphere is the  $\subseteq$ -least receptacle that includes each of Easterner and Westerner as subregions. Therefore

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<sup>21</sup>This is because the closure of the union of Easterner and Westerner includes all of its inner boundary points and hence coincides with Sphere. But since Sphere is by hypothesis an open domain, its interior is Sphere.

SPHERE, if exists, is the least upper bound of EASTERNER and WESTERNER with respect to the part-whole relation as governed by (3).

There is a perfectly parallel result for closed domains:

CLOSED DOMAIN MEREOLGY:

Let  $\mathcal{M}$  be a non-null family of closed domains of a topological space. And suppose  $\mathcal{M}$  is such that (i) the closure of the interior of the union of each non-null family of elements of  $\mathcal{M}$  is itself in  $\mathcal{M}$ ; (ii) if  $A$  and  $B$  are members of  $\mathcal{M}$  such that  $A - B$  is non-empty, then the closure of the interior of  $A - B$  is in  $\mathcal{M}$ . Then  $(\mathcal{M}, \subseteq)$  is a model of classical extensional mereology with respect to which the operation of mereological fusion is interpreted by the function that assigns to each non-null family of closed domains in  $\mathcal{M}$  the closure of its union.<sup>22</sup>

As a special instance of this observation, let  $\mathcal{C}$  be the family of all closures of non-null open domains of Euclidean three-dimensional space. Since  $\mathcal{C}$  is again closed under operations (i) and (ii),  $(\mathcal{C}, \subseteq)$  is a model of (1) – (3).<sup>23</sup> The model, however, does not satisfy (4). For there are pairs of closed domains whose intersection is a non-null two- or one-dimensional region which nevertheless fail to overlap in the model due to the fact that they fail to include another closed domain as a common part. However, it is not difficult to verify that it is a model of (6) as well.

The other difference with respect to the former model is that the family of all closed domains under the subset relation is not a model of (5) or (8) but rather:

- (9) Let  $X$  be the mereological fusion of a non-null family  $\mathcal{F}$  of material objects. Then  $X$  exactly occupies the closure of the interior of the set-theoretic union of the regions of space exactly occupied by the material objects in the family  $\mathcal{F}$ .

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<sup>22</sup>It follows that the operation of mereological complement is interpreted by the function that assigns to each open domain the interior of the closure of its set-theoretic complement.

<sup>23</sup>It may be of interest to note that this model is isomorphic to the model  $(\mathcal{O}, \subseteq)$  described above, where  $\mathcal{O}$  is the family of all open domains of Euclidean space.

And this suffices to show that a closed domain theorist need not be committed to all of the premises invoked in the argument of the last section.<sup>24</sup>

I have suggested that (4) is independent from the closed domain view and (5) is independent from both the open and closed domain views. Since each principle played a crucial role in the argument from (1) – (5) to ( $\star$ ), moderate views of receptacles have an answer to the challenge raised in the last section.

Without (4), we have no means to take the step from the assumption that a material object  $A$  that occupies a point  $p$  has some proper parts to the claim that a proper part of  $A$  exists that does not occupy  $p$ . For it could be that all proper parts of  $A$  occupy  $p$ . To make this vivid, think of a model whose domain consists of two closed cubes  $A$  and  $B$  with an adjacent side and their mereological fusion  $C$ . And suppose that neither  $A$  nor  $B$  have proper parts. Then:  $C$  would be a material object all of whose proper parts occupy  $p$ . But our model is one in which each of (1), (2), (3) and (5) is satisfied.

Without (5), we have no means to make sure that if  $X$  is the mereological fusion of all parts of a material object  $A$  that do not occupy  $p$ , then  $X$  itself will not occupy  $p$ . For, on the open domain view, for example,  $X$  will occupy the interior of the closure of the set theoretic union of all regions of space exactly occupied by all of the parts of  $A$  and this could well contain  $p$  as a member.

**5. Further Challenges.** We have seen that two moderate views of receptacles are live answers to the question of receptacles provided we exercise some care on the details of how different mereological relations interact with exact location. But a proponent of the open and closed domain views of receptacles must at some point face the arguments recently deployed in (Bays, 2003) and (Hudson, 2002) and (Hudson, 2006). I would like to suggest that the observations from the last section will help provide answers to the

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<sup>24</sup>It may be of interest to note that every open domain mereology is isomorphic to some closed domain mereology and vice versa. If  $\mathcal{M}$  is an open domain mereology, then the function that takes each open domain in  $\mathcal{M}$  to its closure is an isomorphism between  $\mathcal{M}$  and a closed domain mereology  $\mathcal{N}$  which consists of all of the closures of members of  $\mathcal{M}$ . Conversely, if  $\mathcal{N}$  is a closed domain mereology, then the function that takes each closed domain in  $\mathcal{N}$  to its interior is an isomorphism between  $\mathcal{N}$  and an open domain mereology  $\mathcal{M}$  which consists of all of the closures of members of  $\mathcal{N}$ .

challenges raised by the recent literature on receptacles. I have postponed a discussion of the argument offered in (Bays, 2003) to an appendix.

Before we look more closely at the credentials of (4) and (5), it might be helpful to briefly explain how to extend the remarks of the last section to deal with some of the arguments deployed in (Hudson, 2002) and (Hudson, 2006). One argument begins with an incontrovertible observation:<sup>25</sup>

If  $R$  is an open continuous region, then  $R$  is *not* the union of a family of pairwise disjoint open regions.<sup>26</sup>

But, the argument begins, if  $R$  is an open continuous region exactly occupied by a material object  $A$ , then either (i)  $A$  is an extended simple or (ii)  $A$  has a partition into pairwise discrete proper parts. (i) is excluded by (6) above. Therefore (ii) must be the case. But since, by (4), pairwise discrete parts must occupy pairwise disjoint regions, Hudson's observation yields that  $A$  has at least one proper part that is not open, pace the thesis that all material objects are open.

If successful, the argument would refute the open domain view of receptacles. But it should by now be apparent how it tacitly relies on (5). For it presupposes that if  $\mathcal{F}$  is a family of material objects each of which exactly occupy each of a family of pairwise disjoint regions, then its mereological fusion should exactly occupy the union of the regions exactly occupied by the members of  $\mathcal{F}$ . But as noticed above, a proponent of the open domain view would be well-advised to replace (5) with (8) in which case the mereological fusion of the members of  $\mathcal{F}$  should exactly occupy not the union but rather the interior of the closure of the union of the regions exactly occupied by the members of  $\mathcal{F}$ . But then: what would be needed to refute the open domain view, for example, is not quite Hudson's observation but rather the claim:

If  $R$  is an open continuous region, then  $R$  is *not* the interior of the closure of a

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<sup>25</sup>In (Hudson, 2002) and (Hudson, 2006), the argument is directed against moderate views on which all receptacles are open.

<sup>26</sup>Two regions  $R$  and  $S$  are *separated* if and only if (i) the intersection of  $R$  with the closure of  $S$  is null, and (ii) the intersection of  $S$  with the closure of  $R$  is null.  $R$  is *discontinuous* if and only if  $R$  is the union of two separated regions.  $R$  is *continuous* if and only if  $R$  is not discontinuous.

union of a family of pairwise disjoint open regions.

But we have every reason to think that this claim is false. Take an open continuous sphere and consider its partition into two disjoint open hemispheres separated by a two-dimensional region. The sphere again is the interior of the closure of the union of the two hemispheres.

The situation is subtler with Hudson's arguments against closed views generally, and the closed domain view in particular.<sup>27</sup> One argument begins with the momentary assumption that there are material objects extended in all spatial dimensions and relies on another incontestable observation:

If  $R$  is an closed continuous region, then  $R$  is the union of a family of pairwise disjoint closed regions only if some of them are themselves two- or one- or zero-dimensional regions.

But, the argument begins, if  $R$  is a closed continuous region exactly occupied by a material object  $A$ , then either (i)  $A$  is an extended simple or (ii)  $A$  has a partition into pairwise discrete proper parts. (i) is excluded by (6) above. Therefore (ii) must be the case. But, by (4), pairwise discrete parts exactly occupy pairwise disjoint regions. By Hudson's observation, however, we are invited to infer that at least one proper part that is not three-dimensional. But similar reasoning takes us from two-dimensional material objects to one-dimensional ones; and from one-dimensional material objects to zero-dimensional ones. But once one grants the existence of point-sized material objects, (2) should enable us to justify the existence of open (and partially open) material objects.

However, by itself, the observation that Hudson's argument implicitly relies on (5) will not help very much. For the problem persists even if one responds that when (9) is substituted for (5), what is needed to refute the closed domain view is not quite Hudson's observation but rather:

If  $R$  is a closed continuous region, then  $R$  is the closure of the interior of the union

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<sup>27</sup>The closed domain view is not the explicit target of the argument in (Hudson, 2002) and (Hudson, 2006), but, if sound, it would clearly tell against it.

of a family of pairwise disjoint closed regions only if some of them are themselves two- or one- or zero-dimensional regions.<sup>28</sup>

While this is not generally true in the infinite case, the closed domain view's foe need only point out that a certain restricted claim is true:

If  $R$  is a closed continuous region, then  $R$  is *not* the closure of the interior of the union of a *finite* family of pairwise disjoint closed regions.

And this observation yields a simple variation on Hudson's original argument: if  $R$  is a closed continuous region exactly occupied by a material object  $A$ , then either (i)  $A$  is an extended simple or (ii)  $A$  has a partition into pairwise discrete proper parts. (i) is excluded by (4) above. Therefore (ii) must be the case. But if (ii) is the case, then  $A$  surely has a partition into *finitely many* pairwise discrete parts. But, by (4), pairwise discrete parts exactly occupy pairwise disjoint regions, which is precisely what appears to be excluded by our last observation.<sup>29</sup>

But the closed domain theorist will not be deterred by this. For the argument makes crucial use of (4) and the closed domain view need not make room for this claim. On the closed domain view, pairwise discrete material objects need not occupy pairwise disjoint regions of space. Take two cubes each of which, let us assume, exactly occupies one of two closed domains with an adjacent side. The two cubes are discrete. This is because, by (3), for a material object to be part of the two cubes, it would have to exactly occupy a subregion of both. But since no closed domain is a subregion of both closed domains, by the closed domain view, no material object that is part of both cubes can exist. Nonetheless, the regions of space exactly occupied by the two cubes are *not* disjoint because their intersection is a two-dimensional region.

Not that the closed domain view is compatible with (5). For (5) would entail that when properly arranged, the fusion of a countable family of pairwise discrete parts, each of which

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<sup>28</sup>By "disjoint" I mean set-theoretically disjoint. I have reserved the term "discrete" for "mereologically discrete."

<sup>29</sup>The alternative would be to maintain that while  $A$  has a partition into infinitely many parts,  $A$  is not the fusion of finitely many parts. While perfectly coherent, it is unclear what the rationale for such a view could be.

exactly occupies a closed domain, can nonetheless exactly occupy a partially open region of space.<sup>30</sup> To make this vivid, consider a Zeno-like series of spherical material objects laid out on an eight feet stretch of a segment with two endpoints:  $p$  and  $q$ . At  $q$ , which is 8 feet from  $p$ , there is the surface of a 16-inch diameter closed sphere that is centered at a point between  $p$  and  $q$  that lies 8 inches from  $q$ . At the point between  $p$  and  $q$  that is 4 feet from  $p$  lies the center of a 8-inch diameter closed sphere. At the point between  $p$  and  $q$  that is 2 feet from  $p$  lies the center of a 4-inch diameter closed sphere, etc. There are infinitely many closed spheres between  $p$  and  $q$  all of which exactly occupy one of a family of pairwise disjoint closed continuous regions. Moreover, for each sphere there are infinitely many spheres closer to  $p$  than that sphere.

What is important for our purposes, however, is the observation that  $p$  is a boundary point of the union of the regions exactly occupied by the spheres in the sequence. This is because any sphere about  $p$  contains points occupied by one closed sphere or another as well as points not occupied by any closed sphere in the sequence —  $p$  itself is not occupied by any closed sphere. Thus (5) yields as a consequence that  $p$  is not occupied by the mereological fusion of all of the spheres in the sequence. And while all the spheres exactly occupy closed regions, their fusion exactly occupies a partially open region.

A closed domain theorist armed with (9) might reply that, despite appearances, the mereological fusion of all of the spheres in the sequence *does* occupy  $p$  because  $p$  lies in the closure of the interior of the union of all of the regions exactly occupied by the spheres in the sequence. Admittedly, this has the rather strange consequence that the fusion of the spheres has as a boundary point one that is not a boundary point of any of the spheres we started with. This is no doubt a cost, but it is quite different from the objection that when properly arranged, the fusion of a family of countably many pairwise disjoint closed domains can still be partially open.

**6. Mereological Fusion and Location.** It is time to take stock. While (3), (1) and (2) do not by themselves entail atomism, (3), (1), (2), (4) and (5) do. And while (3), (1),

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<sup>30</sup>I owe this consideration to Hud Hudson who makes basically this argument in (Hudson, 2001) and (Hudson, 2006).

(2) and (6) do not by themselves entail the existence of single-point material objects, (3), (1), (2), (4), (5) and (6) do. In other words, by itself, the hypothesis that the part-whole relation on material objects is a spatial relation governed by (3) and subject to the axioms of classical extensional mereology provides little guidance when it comes to the question of what regions of space are receptacles. It is only when this hypothesis is supplemented with further assumptions on the interaction of other mereological relations and exact location that we are in a position to discriminate further among different views of receptacles. Two principles, (4) and (5), give voice to two such rather common assumptions.

I will have little to add to the discussion of (4) since different moderate views of receptacles recommend different attitudes towards it. While open domain theorists will no doubt embrace (4) without reservation, closed domain theorists should probably reject it as patently false in view of the counterexamples mentioned above.

More subtle, it seems to me, is the case of (5), which both moderate views are committed to reject. Unfortunately, (5) enjoys a great deal of initial plausibility; indeed, to some, it might seem extravagant to deny it. But that, by itself, is far from a decisive argument for (5), much less a reason to conceive of it as akin to (3) as a plausible a priori constraint on the interaction of the part-whole relation and location. Quite the contrary, the fact that in combination with (3), (1), (2), and (4), (5) delivers a remarkably simple argument for atomism should give us some pause.

The purpose of this section is to carefully examine the credentials of (5) with an eye to the question of whether (5) is a negotiable constraint on the interaction of the relation a mereological fusion of some material objects bears to them and exact location. In what follows, we will explore two different routes to (5). One will be based on the relationship between the mereological structure of material objects and the mereological structure of space itself. The other will be based on the general thought that material objects should inherit their location from their proper parts. While ultimately unsuccessful, each attempt will shed some light on the reasons why the principle might strike us as initially plausible and the question of whether it bears closer scrutiny. As we look more closely at the two candidate motivations for the principle, I hope to convince you that each motivation

either requires further substantive premises that are not beyond doubt or justifies a weaker principle in the vicinity of (5) from which this cannot be derived as a consequence. I suspect this weaker principle is what underlies the apparent intuitive plausibility of (5).

Let me begin with considerations based on the mereological structure of space itself. Assume that substantivalism is true and that there are regions of space best represented as sets of points of ordinary Euclidean space. Once one countenances regions of space, it is not uncommon to take regions of space themselves to be parts of other regions — or at least not when regions are identified with sets of points. The part-whole relation on regions of space is generally identified with the subset relation:  $R$  is part of  $S$  if and only if  $R$  is a subset of  $S$ . In the presence of this identification, we have that two regions  $R$  and  $S$  overlap if and only if they have a non-null intersection. Moreover, we have that the mereological fusion of a family of regions of space  $\mathcal{R}$  is its set-theoretic union  $\bigcup \mathcal{R}$ .

But one might be tempted to assume, in addition, that mereological relations on material objects are perfectly aligned with their counterparts on the regions of space exactly occupied by the material objects in question. Thus two material objects overlap if and only if the regions of space they exactly occupy overlap, i.e., have a non-null intersection. Likewise, a material object is the mereological fusion of a family of material objects if and only if the region of space exactly occupied by the first is the mereological fusion of all the regions of space exactly occupied by the material objects in the family. In other words, if  $\mathcal{A}$  is a family of material objects and  $\mathcal{R}$  is the family of regions of space exactly occupied by them, then one might have expected the mereological fusion of material objects in  $\mathcal{A}$  to exactly occupy the mereological fusion of regions in  $\mathcal{R}$ , i.e.,  $\bigcup \mathcal{R}$ .

In sum, when combined with the assumption that mereological relations on material objects are perfectly aligned with mereological relations on the regions of space they exactly occupy, the identification of the part-whole relation on regions of space with the subset relation provides us with a rationale for each of (4) and (5). But why expect such a remarkable alignment in the first place?

One reason might stem from the identification of material objects with the regions of space they exactly occupy. Against the background of substantivalism, the question im-

mediately arises of what exactly is the relation between material objects and the regions of space they exactly occupy. While it is common to think of material objects as different from the regions of space they exactly occupy, one could adopt a different view on which they are in fact identical to them — this is what is sometimes known as supersubstantivalism.<sup>31</sup> Since exact occupation is, for the supersubstantivalist, identity, it might be suggested that the mereological fusion of some material objects exactly occupies the fusion of their receptacles; they are one and the same.

It is not clear that the letter of supersubstantivalism entails (4) and (5). For while all material objects are identified with regions of space on this view, presumably not all regions of space count as material objects. It would not be unreasonable to identify material objects with regions of space that exemplify a certain intrinsic property whereby they count as suitably matter-filled. But there might be restrictions on the sorts of regions of space that can be appropriately matter filled, in which case supersubstantivalism would be compatible with the claim that not all regions of space are or could be material objects. Indeed, the letter of supersubstantivalism seems to me to be consistent with the claim that only open domains, for example, could be appropriately matter filled, and, a fortiori, candidate material objects. This, however, would undermine the support for (5). Likewise for the claim that only closures of non-null open domains could be appropriately matter filled, which would undermine the support for (4).

The letter of supersubstantivalism may not entail (4) or (5) but the spirit of supersubstantivalism might be thought to be in line with it. For it may seem plausible to assume further that any subregion of a matter filled region of space is itself matter filled.<sup>32</sup> And this further thought yields as an immediate consequence what we may call a principle of arbitrary partition:

- (10) ARBITRARY PARTITION: For any material object  $x$ , if  $R$  is a non-null subregion of the region of space exactly occupied by  $x$ , then there is some material object  $y$  such

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<sup>31</sup>The term *supersubstantivalism* is used, for example, by Lawrence Sklar in (Sklar, 1974). I'm indebted to Hud Hudson for suggesting supersubstantivalism as a rationale for (5).

<sup>32</sup>Similar claims are made in (Sider, 2003) and (Parsons, 2006).

that (i)  $y$  is part of  $x$  and (ii)  $y$  exactly occupies  $R$ .<sup>33</sup>

But there is, in the presence of (3), a swift argument from (10) to each of (4) and (5). While, to be sure, by the lights of supersubstantialism, exact location is identity, it should be noticed that, once (10) has been motivated, there is no need to invoke that identification again in the course of the arguments. In other words, in the presence of (3), (10) gives us all we need for a defense of (4) and (5).

Take the case of (4). One direction is trivial and requires no use of (10). If  $x$  and  $y$  overlap, then they have some part in common  $z$ , which, by (3), must exactly occupy a region all of whose points are occupied by each  $x$  and  $y$ . Therefore,  $x$  and  $y$  must have a non-null intersection. The other direction is more delicate. Suppose that the regions of space exactly occupied by  $x$  and  $y$  have a non-null intersection. Then, by (10), this non-null intersection must be exactly occupied by some material object which is part of each of  $x$  and  $y$ .

Take now the case of (5). Let  $\mathcal{A}$  be a family of material objects and let  $\mathcal{R}$  be the family of regions of space exactly occupied by them. By (3), we have that the mereological fusion of  $\mathcal{A}$  includes each member of  $\mathcal{R}$  as a subset and therefore includes  $\bigcup \mathcal{R}$  as a subregion. But, by (10),  $\bigcup \mathcal{R}$  itself must be exactly occupied by a material object, which, by definition of *fusion*, will qualify as the mereological fusion of  $\mathcal{A}$ .

Where does this leave us? It seems that some philosophers may have principled reasons to accept (4) and (5) after all, provided they share other substantial theoretical commitments with respect to the ontology of space and the identification of material objects with the regions of space they exactly occupy. More generally, the last two arguments tell us, in fact, that (4) and (5) will be available provided (10) is. This observation, however, has only limited value for those of us without the theoretical commitments in question. Some of them, in fact, will be quite unpalatable for a philosopher moved by Cartwright's quote. Take (10) for example. If true, (10) would tell us that if there are material objects, then there are single-point material objects or material objects which manage to exactly occupy a finite or, in some cases, a countable number of points.

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<sup>33</sup>I borrow the label "arbitrary partition" from (Parsons, 2006).

There is, however, an alternative route to (5) which, if successful, would require minimal theoretical commitments and should have some force for all parties involved in the debate. This route involves an appeal to the intimacy of the part-whole relation on material objects. This feature of the part-whole relation is what underlies different forms of what has come to be known as Composition as Identity, a family of views on which the relation between a whole and its proper parts is claimed to be a particularly intimate relation akin to identity: a whole, we are invited to agree, is “nothing over and above” its parts. In its most extreme form, Composition as Identity just is the thesis that a mereological fusion is identical with its parts. Unfortunately, the thesis that a whole is in fact identical to its parts is highly controversial, if not unintelligible. Much more plausible is the view that composition, the relation some proper parts bear to a whole, is analogous to identity in various respects. What is important for our purposes is the question of whether (5) might be argued to fall out of this attractive picture of the part-whole relation on which a whole is “nothing over and above” its parts.<sup>34</sup>

However one specifies the respects in which composition is supposed to be akin to identity, one would have expected one such respect to concern the relation between the location of a whole and the location of its proper parts. Just as an object is located exactly wherever any identical object identical is located, a whole should be located exactly wherever its parts are located. This is what one might attempt to express with the help of the principle:

- (11) INHERITANCE OF LOCATION: If a mereological fusion has some proper parts, then it occupies a point  $p$  if and only if some proper part of it occupies  $p$ .<sup>35</sup>

But it would seem to follow from this that a mereological fusion exactly occupies the set-theoretic union of the regions of space exactly occupied by all of its proper parts:

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<sup>34</sup>Ted Sider offers a defense of moderate forms of Composition as Identity in (Sider, ming).

<sup>35</sup>We need the qualification that the fusion has proper parts to make sure the principle does not collapse into an utterly trivially claim like, for example:

A mereological fusion occupies a space point  $p$  if and only if  $p$  is occupied by some of part of it.

The problem of course is that anything being an improper part of itself, a mereological fusion will occupy a point  $p$  if and only if  $p$  is occupied by its improper part.

- (12) MODERATE FUSIONS: Let  $X$  be the mereological fusion of a non-null family  $\mathcal{F}$  of material objects other than  $X$ . Then  $X$  exactly occupies the set-theoretic union of the regions of space exactly occupied by *all* of its proper parts — whether or not they are members the family  $\mathcal{F}$ .

Since (12) has been independently motivated by the intimacy of the part-whole relation, one might hope this motivation to carry over to the case of (5) in which case we would have again have the rudiments for a powerful argument for atomism, in the first place, and the liberal view of receptacles. This would happen, for example, if a case could be made that (12) entails (5) because failures of (5) would immediately give rise to failures of (12).

Perhaps surprisingly, (5) is independent from (12). To appreciate this, note first that failures of (5) need not lead to failures of (12). Let SPHERE be the mereological fusion of two material objects: EASTERNER and WESTERNER. Let Sphere be the open domain exactly occupied by SPHERE. And let Easterner and Westerner be two open domains exactly occupied by EASTERNER and WESTERNER, and which correspond to two hemispheres of Sphere. We have a failure of (5) because Sphere is not the set-theoretic union of Easterner and Westerner, which are separated by a two-dimensional region that is included in Sphere. This, however, need not give rise to a counterexample to (12). For whenever  $p$  is a point in the two-dimensional region that separates Easterner from Westerner, we are in a position to find a proper part of SPHERE which is neither Easterner nor Westerner but nevertheless occupies  $p$ . For example, if  $p$  lies in the northern hemisphere of Sphere, which we may call Northerner, we could presumably pick a proper part of SPHERE, NORTHERNER, which exactly occupies  $p$  despite its not being one of EASTERNER and WESTERNER.

We are now in a position to turn these suggestive considerations into a more precise argument to the effect that (12) does not entail (5) even against the background of (3). I would like to make it plausible to assume that, modulo certain assumptions, it is possible for (3) and (12) to be true while (5) remains false.<sup>36</sup> To that purpose, we need only assume that two hypotheses are in fact possible:

OPEN DOMAIN VIEW: A region of space is a receptacle if and only if it is a non-null

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<sup>36</sup>I am indebted to Ignacio Jané for discussion.

open domain.

DOCTRINE OF ARBITRARY UNDETACHED PARTS: If  $x$  is a material object and  $R$  is a receptacle included in the region of space exactly occupied by  $x$ , then there is some material object  $y$  such that (i)  $y$  is part of  $x$  and (ii)  $y$  exactly occupies  $R$ .

For suppose that  $x$  is a mereological fusion of a non-null family of material objects other than  $x$ . Then, on the open domain view,  $x$  exactly occupies an open domain  $R$ . Now: if  $p$  is a point in  $R$ , say that an open domain  $O$  is of the form  $O_p$  if and only if  $O$  is a proper subset of  $R$  and  $O$  contains  $p$  as a member. Two observations:

- (i)  $y$  is a proper part of  $x$  if and only if  $y$  exactly occupies some proper subregion of  $R$  which is a non-null open domain of the form  $O_p$  for some  $p$  in  $R$ .
- (ii) If  $p$  is a member of an open domain  $R$ , there is some proper subregion of  $R$  which is a non-null open domain of the form  $O_p$ .

While (i) is justified by appeal to (3) and the open domain view, (ii) is just a plain geometrical fact. The key observation now is that if  $R$  is an open domain, then  $R$  is the union of all  $O_p$  for  $p \in R$ . (This is because, by (ii), every point  $p$  in  $R$  is a member of some  $O_p$  and every member of every  $O_p$  is a member of  $R$ .) But, by (i),  $x$  is the mereological fusion of all and only material objects of the form  $y$  where  $y$  exactly occupies some open domain  $O_p$  for some  $p$  in  $R$ . Therefore both (3) and (12) are true. However, it is not difficult to adapt the examples discussed above to find a partition of  $x$  into two different material objects  $y$  and  $z$  such that the region of space exactly occupied by  $x$  is not the set-theoretic union of the regions of space exactly occupied by  $y$  and  $z$ . Thus (5) fails.

I conclude that even if we take the intuitive plausibility of (12) at face value, by itself, (12) lends no plausibility whatever to (5). I would dare conjecture that the reason we initially think of (5) as obvious is because pre-theoretically we tend to conflate the two. Once we are in a position to discern the differences between the two, we have no reason to regard (5) as equally plausible.

**7. Conclusion.** I have suggested that neither atomism nor the liberal view of receptacles is forced upon one by one's allegiance to (1), (2), and (3). The situation is importantly different for someone who is prepared to accept, in addition, (4) and (5). And the question emerged of whether there is some independent reason to think of (4) and (5) as non-negotiable constraints on the interaction of various mereological relations with location akin to (3). I focused on (5) and found two candidate lines of defense to either rely on substantial theoretical commitments without independent support or to fail to deliver the necessary support for the principle.

Fortunately, a little reflection on the central argument of the paper affords us with a further reason to reject (4) and (5) as necessary constraints on the interaction of various mereological relations with exact location — much like (1), (2) and (3) would appear to be. The reason is not difficult to explain. While one might find it plausible to assume each of (1), (2), and (3) are necessary truths, if true, their combination with (4) and (5) would, by the argument outlined in section 3, immediately deliver atomism as a consequence. But if (4) and (5) are taken to be necessary, then we have an argument not only for atomism but even for the necessity of atomism. Yet, it seems genuinely possible that there could be atomless gunk. This could happen in a world in which one of (8) or (9) is true and (5) is false. But such worlds strike me as genuine possibilities.

I do not, however, claim to have provided independent reasons to accept one or another moderate view of receptacles. What I hope to have shown is that as far as classical extensional mereology and the interaction of various mereological relations with exact location are concerned, moderate views of receptacles are still live answers to Cartwright's original question. What sorts of considerations might help us make further progress and finer discriminations is a question for another occasion.

**Appendix.** (Bays, 2003) would seem to suggest that (1), (2), (3) and (6) entail the existence of single point receptacles.<sup>37</sup> The argument relies on a transfinite recursion on

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<sup>37</sup>In private communication, however, Timothy Bays has pointed out to me that in (Bays, 2003) he had taken "overlap" to mean *spatial overlap*, where  $x$  and  $y$  spatially overlap if and only if there is a point they both occupy, and not *mereological overlap* as generally defined in terms of *part*:  $x$  and  $y$  overlap if and only if they have a common part. This makes his formulation of (1) differ in strength from the mereological

the ordinals and therefore involves a tacit appeal to the axiom of choice. It asks us to consider an arbitrary material object  $A$  and a point  $p$  it occupies and it invites us to conclude that  $\{p\}$  is a receptacle for some part of  $A$ . For let us assume that  $\{p\}$  is a proper subset of the region exactly occupied by  $A$ . Then one may construct a sequence of proper parts of  $A$  indexed by ordinals:  $A_0 < A_1 < A_2 < \dots < A_\beta < A_{\beta+1} < \dots$ , where each  $A_\beta$  in the sequence is a proper part of  $A_{\beta+1}$ . In order to specify the sequence, we need to distinguish three cases:

- $A_0$ : Since, by (6),  $A$  is not an extended simple, there is a proper part  $B$  of  $A$ . If  $B$  does not occupy  $p$ , then let  $A_0 = B$ ; otherwise, use (2) to obtain another proper part  $B'$  of  $A$  that is discrete from  $B$ . Since  $B'$  is discrete from  $B$  and  $B$  occupies  $p$ ,  $B'$  does not, in which case we may let  $A_0 = B'$ .
- $A_{\beta+1}$ : Given  $A_\beta$ , use (1) again to obtain a proper part  $B_\beta$  of  $A$  that is discrete from  $A_\beta$ . There are three cases:
  - Case 1:  $B_\beta$  does not occupy  $p$ . Then let  $A_{\beta+1} = A_\beta + B_\beta$ , e.g., the mereological fusion of  $A_\beta$  and  $B_\beta$ .
  - Case 2:  $B_\beta$  occupies  $p$  but  $\{p\}$  is a proper subset of the region exactly occupied by  $B_\beta$ . By (4)  $B_\beta$  is not an extended simple and there is some proper part  $C_\beta$  of  $B_\beta$ . If  $C_\beta$  does not occupy  $p$ , let  $A_{\beta+1} = A_\beta + C_\beta$ ; otherwise, use (1) to obtain a proper part  $C'_\beta$  of  $B_\beta$  that is discrete from  $C_\beta$ . Since  $C_\beta$  occupies  $p$ ,  $C'_\beta$  does not, in which case we may let  $A_{\beta+1} = A_\beta + C'_\beta$ .
  - Case 3:  $B_\beta$  exactly occupies  $\{p\}$ . Then  $\{p\}$  is a receptacle for  $B_\beta$  and one is done.
- $A_\lambda$ , where  $\lambda$  is a limit ordinal: Let  $A_\lambda$  be the mereological fusion of all  $A_\alpha$  for  $\alpha < \lambda$  — which exists by (2).

But then, the argument continues, the receptacles of the proper parts of  $A$  in the sequence  $A_0 < A_1 < A_2 < \dots < A_\beta < A_{\beta+1} < \dots$  form a constantly increasing sequence. For principle of Weak Supplementation, which is directly concerned with mereological overlap as defined above. (4) provides a route from the mereological principle to Bays's principle.

cardinality reasons, however, the construction must terminate in fewer than  $(2^{\aleph_0})^+$  steps at some Case 3 situation. Hence there must be some proper part  $A_\beta$  of  $A$  with  $\{p\}$  as its receptacle.<sup>38</sup>

However, if for two material objects to overlap is just for them to have a part in common, then the argument requires us to assume (4) and (5). Without (4), we would not be licensed to move from the claim that two parts of  $A$  are discrete to the further claim that they exactly occupy disjoint regions of space. This step is taken in the definition of the 0 and successor stages of the transfinite recursion.

More importantly, without (5), we lack the assurance that the construction will not peter out at some limit level of the hierarchy. For even if a point  $p$  is not occupied by any  $A_\beta$  for  $\beta$  strictly less than some limit ordinal  $\lambda$ ,  $p$  might still be a member of the interior of the closure of the union of all the regions of space exactly occupied by all  $A_\beta$  for  $\beta < \lambda$ . But, in the absence of (5), we have no guarantee that the mereological fusion of all the  $A_\beta$  for  $\beta < \lambda$  will not exactly occupy the interior of the closure of the union of all the regions of space exactly occupied by all  $A_\beta$  for  $\beta < \lambda$ .<sup>39</sup>

## References

- Bays, T. (2003). Hudson on receptacles. *Australasian Journal of Philosophy*, 81(4):569–572.
- Cartwright, R. (1975). Scattered objects. In Lehrer, K., editor, *Analysis and Metaphysics*, pages 153–171. Reidel, Dordrecht. Reprinted in (Cartwright, 1987), 171–186.
- Cartwright, R. (1987). *Philosophical Essays*. MIT Press, Cambridge, MA.
- Hawthorne, J. and Weatherson, B. (2004). Chopping up gunk. *The Monist*, 87:339–350.
- Hudson, H. (2001). Touching. *Philosophical Perspectives*, pages 119–128.

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<sup>38</sup>This is because at most  $2^{\aleph_0}$  points occupied by  $A$  remain to be occupied by further receptacles at any stage in the sequence.

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- Hudson, H. (2002). The liberal view of receptacles. *Australasian Journal of Philosophy*, 80(4):432–439.
- Hudson, H. (2006). *The Metaphysics of Hyperspace*. Oxford University Press.
- Lewis, D. (1991). *Parts of Classes*. Blackwell, Oxford.
- Parsons, J. (2006). Theories of location. In *Oxford Studies in Metaphysics*. Oxford University Press.
- Sider, T. (2003). *Four-Dimensionalism: An Ontology of Persistence and Time*. Oxford University Press, Oxford.
- Sider, T. (forthcoming). Parthood. *The Philosophical Review*.
- Sklar, L. (1974). *Space, Time and Spacetime*. University of California Press.
- Tarski, A. (1929). Foundations of the geometry of solids. In *Logic, Semantics and Meta-Mathematics*, pages 24–29. Hackett. In Alfred Tarski, *Logic, Semantics and Meta-Mathematics*.
- Whitehead, A. N. (1929). *Process and Reality*. The Mac Millan Company.