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Vagueness: A Global Approach, by Kit Fine. Oxford: Oxford University Press, 2020. Pp. xx + 100.

This pocket-sized volume derives from Kit Fine's three 2016 Rutgers Lectures in Philosophy, which inaugurated the annual series. He has already published some of the material in two articles, but it is useful to have a unified exposition. As one would expect from Fine, the discussion is highly original, despite the well-trodden ground, technically ingenious and well-developed, and inspired by a big picture. Whether his approach will catch on remains to be seen.

The first part of the book is introductory and largely familiar, sketching three mainstream views, Degree-ism, Supervaluationism, and Epistemicism, and Fine's objections to them Although he authored the classic case for Supervaluationism (Fine 1975, still his most-cited work), he has long been dissatisfied with the view; we now have his attempt to do better.

The key idea is that vagueness is a kind of indeterminacy which is *global* rather than *local*. 'Local indeterminacy is indeterminacy in the application of the predicate to a single object. … Global indeterminacy, by contrast, is indeterminacy in the application of the predicate to a *range* of cases' (p. 18). So, with vagueness, there is no such thing as an isolated borderline case. Technically, the contrast is manifested by a logic in which one cannot deny a single instance of the law of excluded middle ($\neg(p \vee \neg p)$ is inconsistent), but one *can* deny some conjunctions of two or more instances of the law ($\neg((p \vee \neg p) \wedge (q \vee \neg q))$ is consistent). Thus a sorites series exhibits vagueness although no member of it does so by itself.

Fine aims to characterize the indeterminacy of the sorites series in a way compatible with applying the predicate (such as 'bald') at one end and denying it at the other, but incompatible with any *sharp* characterization, one which describes two successive members in mutually incompatible terms. That is his version of Mark Sainsbury's claim that 'Vague concepts are concepts without boundaries' (Sainsbury 1989). He proves that under very general conditions, in a standard propositional language with a 'definitely' operator, there is no such characterization of indeterminacy (Appendix A, 'The Impossibility Theorem').

To meet the challenge, Fine drops the 'definitely' operator and gives a non-classical model theory for the other operators. A model is a triple of a nonempty set of points of evaluation ('uses'), a reflexive and symmetric binary relation between uses ('compatibility'), and a mapping of each sentence letter to the set of uses at which it is true. His semantic clauses for the operators are these, where *u* is a use and B and C are formulas (pp. 42, 79-80):

(i) $B \wedge C$ is true at *u* iff B is true at *u* and C is true at *u*.
(ii) $B \vee C$ is true at *u* iff B is true at *u* or C is true at *u*.
(iii) $\neg B$ is true at *u* iff no use at which B is true is compatible with *u*.
(iv) $B \supset C$ is true at *u* iff either B and C are true at *u* or C is true at every use compatible with *u* at which B is true.

As usual, a formula C is a logical consequence of a set of formulas Γ ($\Gamma \vDash C$) iff, in every model, C is at true at every use at which every member of Γ is true. Fine shows that the resultant logic provides a characterization of indeterminacy meeting his desiderata; it is the negation of a conjunction of instances of excluded middle (Appendix B, 'The Possibility Theorem').

An intriguing final chapter applies the account to several versions of the sorites paradox, to the reviewer's anti-luminosity argument (Williamson 2000: 96-97), and to Derek Parfit's treatment of fission cases for personal identity.

Many aspects of Fine's approach merit discussion.

Local or global?

Fine's characterization of his account as 'global' is somewhat misleading. The globe in question can be very small indeed. Consider Fine's patch Pat 'on the border between red and orange'; as he says, 'red and orange are exclusive colors' (p. 10). Let Rp and Op formalize 'Pat is red' and 'Pat is orange' respectively. Then $\neg((Rp \vee \neg Rp) \wedge (Op \vee \neg Op))$ is consistent in Fine's logic, for it is true in a model isomorphic to one Fine himself uses (p. 42). For future reference, call the model **M**; it has just two uses, which are compatible; Rp is true just at one and Op just at the other, so R and O are mutually exclusive, in the sense that they are nowhere both true. Thus $\neg Rp$ is true at neither use, since each is compatible with the one where Rp is true; hence $Rp \vee \neg Rp$ is true only at the use where Rp is true. Analogously, $Op \vee \neg Op$ is true only at the use where Op is true. Therefore, their conjunction is true at neither use, so its negation is true at both uses. Thus Fine's logic permits indeterminacy for a single object with respect to a pair of penumbrally connected predicates. A slight tweak to the model allows indeterminacy in an even stronger form, where $\neg Rp$ and $\neg Op$ are uniformly substituted for Rp and Op in the formula (compare the four-use model on p. 43). It looks very like local indeterminacy on the traditional conception of a single borderline case: admittedly, two rival predicates are involved, but then every border has two sides.

Comparison with intuitionistic logic

Fine compares his semantics to Kripke's semantics for intuitionistic logic (Kripke 1963), with symmetry replacing transitivity as a constraint on the accessibility relation, and the extra disjunct in (iv) making $B \supset C$ true when both B and C are true (p. 41). He does not mention another difference: in the Kripke semantics, each sentence letter p is required to be upward persistent, in the sense that whenever p is true at a point *u*, and a point *v* is accessible from *u*, p is true at *v* too. The intuitionistic clauses for the operators then ensure that all complex formulas inherit this upward persistence. Informally, information is never lost in moving to an accessible point. Given upward persistence, the extra disjunct would be redundant in the clause for the intuitionistic conditional. By contrast, Fine could not impose the analogue of upward persistence for compatibility on his models, for given the symmetry of compatibility it would mean that exactly the same atomic formulas were true at any two compatible uses, and so likewise for complex formulas: the logic of such models collapses into classical logic.

Fine's logic has many features in common with intuitionistic logic. For instance, double negation behaves in the same way in the two logics: A always entails $\neg\neg A$, but $\neg\neg A$ does not always entail A; $\neg\neg\neg A$ is always equivalent to $\neg A$. Whereas the failure of excluded middle is usual for many-valued logics too, the failure of double negation elimination is not.

However, the differences explained above make Fine's logic much less well-behaved than intuitionistic logic. For example, unlike the intuitionistic conditional, Fine's $\supset$ is non-transitive: $A \supset B, B \supset C \vDash A \supset C$ fails. In model **M**, $Rp \supset (Rp \vee Op)$ is true at both uses, and $(Rp \vee Op) \supset Op$ is true at the use where Op is true, but $Rp \supset Op$ is true at neither use. Thus transitivity fails at the use where Op is true (and analogously at the other use). Moreover, since $A \supset (A \vee B)$ is valid in Fine's logic, it also invalidates the intuitionistically valid schema $(A \vee B) \supset C \vDash A \supset C$. Informally, the argument 'If she is in Amsterdam or Berlin, she is happy; therefore, if she is in Amstrerdam, she is happy' is logically fallacious.

A related difference is that Fine's logic validates only a very restricted version of conditional proof ($\supset$ introduction), with no side premises: from $A \vDash C$ to $\vDash A \supset C$. Intuitionistically, if C is provable from A and a set Γ of side premises, then $A \supset C$ is provable from Γ; that is the canonical way to establish a conditional. By contrast, unrestricted conditional proof fails in Fine's logic. For instance, although the schema $A, B \vDash A \wedge B$ is valid, the schema $A \vDash B \supset (A \wedge B)$ is not. In model **M**, $Rp \supset (Op \wedge Rp)$ fails at the use where Op is true. Informally, the argument 'He is in Amsterdam; therefore, if he is drunk he is drunk in Amsterdam' is logically fallacious.

The differences mentioned so far between the two logics all involve $\supset$. But some differences do not. One concerns an intuitionistically valid form of *reductio ad absurdum*, again with side premises: if an absurdity $\perp$ is provable from A and Γ, then $\neg A$ is provable from Γ. That fails in Fine's logic. For example, we have $\neg(A \wedge B), A, B \vDash \perp$ but not $\neg(A \wedge B), A \vDash \neg B$ (both are intuitionistically valid). In model **M**,$\neg(Rp \wedge Op)$ is true at both nodes, and Rp at one, but $\neg Op$ fails at both. Informally, if you already know that Janet will be at the party, and a reliable source tells you that Janet and John will not both be at the party, in Fine's logic you cannot deduce that John will not be at the party: a disappointing result.

Realism or instrumentalism about the semantics

As the examples above suggest, working in Fine's logic is at best awkward. It continually trips up natural modes of reasoning. Could Fine respond that his semantics explains *why* those modes of reasoning are not strictly valid? That retort involves treating the semantics as explanatory rather than merely instrumental: not simply a mathematical device for specifying a particular consequence relation over formulas, but an account of what those formulas can *really mean*. But there is reason to doubt that Fine is in a position to treat his semantic framework in that realist way. For if meanings are really to be specified by stipulating truth-conditions over a set of uses, then we should be able to specify a second negation operator ~ thus:

(v) ~B is true at *u* iff B is not true at *u*.

For if (i)-(iv) set the standard for what a semantic clause should look like, (v) meets that standard; it is just as good as they are. Indeed, ~ also seems to have a better claim than $\neg$ to be genuine negation, since it corresponds directly to negation ('not') in the metalanguage, on the right-hand side of (v), by contrast with the additional semantic complications on the right-hand side of (iii), the semantic clause for $\neg$. However, with an operator such as ~ in the language, the conditions of Fine's Possibility Theorem vanish, for (v) turns every difference in what is true into an inconsistency. Indeed, Fine considers an operator defined exactly like ~, which he paraphrases as ‘it is not-true that’, and the threat it poses to his position (p. 50). He responds by claiming that ‘there is no concept there’, that the ‘broader concept of not-being-true, as opposed to not being the case [¬], is an illusion’ (p. 51). These claims have no basis in his formal semantic framework. Rather, if he is right, they point to a limitation of that framework, its failure to represent the semantic possibilities perspicuously, because it leaves space for what is not a genuine semantic possibility.

Fine’s objection to (v) as a semantic clause can hardly be that its right-hand side is meaningless. On the contrary, it is perfectly intelligible, by the compositional semantics of English. Indeed, in his proof of the Possibility Theorem, Fine himself repeatedly negates metalinguistic formulas of the form $u \vDash B$ (the formalization of ‘B is true at *u*’), by writing an oblique line through $\vDash$ (pp. 80-82). Instead, the point must be that although (v) itself is meaningful, it does not determine a genuine meaning for ~ because it violates some constraint obscured by the formal framework.

The case of the Kripke semantics for intuitionistic logic is analogous. Committed intuitionists normally deny that it is the intended semantics. The latter is to be given in quite different proof-theoretic terms. If the Kripke semantics were intended, there would be a serious issue about the analogue of (v). For although it makes perfect sense within the formal framework, it would violate upward persistence and thereby undermine the soundness of intuitionistic logic for the extended semantics. Thus the Kripke semantics is relegated to an instrumental role, as a mathematically elegant device for determining whether one formula is an intuitionistic consequence of others.

Fine could appropriately take a similar instrumentalist attitude to his own formal semantics. But having done so, he could not also appeal to the formal semantics to explain or justify the widespread failures of elementary logical principles which he is postulating.

Similar issues arise for the definition of a ‘definitely’ operator, D. Fine’s formal framework allows it to be done:

(vi) D[B] is true at *u* iff B is true at every use compatible with *u*.

This invokes nothing more than (iii) and (iv) do. But Fine will reject (vi), since it explicitly articulates local indeterminacy for individual formulas: ¬D[Rp] ∧ ¬D[¬Rp] holds in model **M**.

Furthermore, D would undermine Fine’s treatment of identity puzzles, by providing the means to express a version of Gareth Evans’ argument for the definiteness of identity (Evans 1978). Fine considers a fission case involving the pre-fission person Primo (P) and the post-fission persons Lefty (L) and Righty (R). His analysis relies on a model whose four uses form a diamond: both identities P = L and P = R are true at the top corner, just P = L is true at the left corner, just P = R is true at the right corner, and neither identity is true at the bottom corner; only opposite corners are incompatible. Thus, although P = L is true at the left corner, by (vi) D[P = L] is not, because P = L is not true at the bottom corner, which is compatible with the left corner. Of course, P = P is true at every corner, so by (vi) D[P = P] is also true at every corner. Thus both P = L and D[P = P] are true at the left corner, so by Leibniz’s Law D[P = L] should also be true at the left corner. So, since the bottom corner is compatible with the left corner, by (vi) P = L should be true at the bottom corner; but by hypothesis it is not. One might resist the application of Leibniz’s Law if one regarded ‘P’ or ‘L’ as a vague name, but that move is quite alien to Fine’s treatment; he treats the issues as logical and metaphysical, not linguistic. Consequently, Fine must deny that (vi) defines a meaningful operator (he informs me that he discusses related issues in Fine 2020). Given the naturalness of the definition in his formal semantic framework, that is another reason for him to take an instrumentalist attitude to the framework.

What metalogic?

Another puzzle about Fine's attitude to his non-classical logic is that in his metalogical proofs he relies without comment on distinctively classical reasoning. For example, in his proof of the Possibility Theorem he argues from a supposition about a sharp response to a statement (#), basically by supposing that (#) is not true and deriving a contradiction under the supposition (p. 85). Thus the overall structure of the argument involves another form of *reductio ad absurdum*, from A, $\neg$C $\vDash \bot$ to A $\vDash$ C, which is invalid in Fine's logic (and in intuitionistic logic). If he rejects classical logic for the object language, how is he entitled to rely on it for the metalanguage?

The standard response to such objections is that if classical logic has been rejected only for reasons of vagueness, it can still be legitimately used in mathematical proofs, since mathematical language is precise. But that response is superficial. For a start, the presence of undecidable statements such as the Continuum Hypothesis in set theory has been thought to render the underlying language of mathematics somewhat vague. Even if we ignore that worry, rejecting classical logic for vague languages but not for the language of mathematics undermines *applications* of pure mathematics to non-mathematical domains. Such applications require us to instantiate universally quantified theorems of pure mathematics, proved using classical logic, by substituting non-mathematical terms for the quantified variables. Those non-mathematical terms may well be vague. Since classical logic is supposed to fail for vague terms, the corresponding instances are unsafe. A trivial example is the theorem of pure classical logic $\forall x\ \forall y\ (x = y \lor x \neq y)$; its instantiation by two vague terms is of the form $a = b \lor a \neq b$, which someone who takes excluded middle to fail in the presence of vagueness may well doubt (see Williamson 2018 for more discussion).

How might these general considerations engage with Fine's metalogic? The Possibility Theorem says (roughly) that his logic makes a formula attributing strong global indeterminacy to a sorites series compatible with the obvious statements at each end of the series but incompatible with every *sharp response* to the series. A sharp response here is, again roughly, a sequence of formulas classifying (in relevant respects) the successive members of the series, not all of them in identical ways, where any two members are classified in either identical or incompatible ways. Fine resists the application of classical logic to matters of identity in his treatment of fission cases; in principle, identity puzzles could also arise for formulas and sequences of formulas, undermining the application of classical logic to them too. Not even the sharpness of a response excludes vagueness with respect to what its members are. Thus it is far from obvious that Fine, by his own lights, is entitled to rely on classical logic in reasoning about all sharp responses. At the very least, he needs to provide some justification for his reliance on a classical metalogic.

Of course, those of us who are comfortable with classical logic even for vague languages can accept Fine's metalogical proofs just as they stand: but Fine himself is not in that happy position.

Methodology

Of responses to the Impossibility Theorem on behalf of older theories, Fine writes (p. 22): 'My own view is that none of these responses to the impossibility result will ultimately stand up; and, if this is so, then all of the existing approaches to vagueness should be abandoned and some other way of evading the result should be found'—presumably, his own. The comment typifies a rather local conception of the appropriate methodology for comparing philosophical theories, evident at various points in the book. The task is to identify each rival theory's fatal flaw, which can then be exploited to eliminate that theory. That is rarely how it works. Especially with longstanding philosophical paradoxes, such as the sorites paradox, it is a pretty safe bet that there is no pain-free thorough understanding of the phenomena. All the extant alternatives have well-known painful aspects, and any alternative not yet thought of is likely to be painfully unnatural. For each of the main views, there will continue to be those who find it less painful than any of its rivals. We certainly should not abandon all the existing approaches to vagueness in the mere hope of finding a better one: all the others may be even more painful.

A more constructive methodology is for proponents of each theory to develop and apply its explanatory resources as far as they can. That may involve integrating it, or at least reconciling it, with theories in neighbouring domains. It is worth mentioning some aspects of the challenge in relation to Fine's theory.

Like other non-classical logics, Fine's logic of vagueness faces the problem that by far the most successful deductive enterprise in history is mathematics, based on classical logic. Although that does not automatically mean that logic outside mathematics must be classical too, if it is not we need to understand how the interface works between classical logic inside mathematics and non-classical logic outside, especially for applications of classical results to (allegedly) non-classical environments. Typically, those making the applications do not recognize the non-classicality of the environment and so take no special precautions, yet in practice the transition seems to cause remarkably few problems. For reasons explained in the previous section, that task is much harder than non-classical logicians tend to realize.

Vagueness is ubiquitous in natural language. The formal semantics of natural languages is a well-developed branch of linguistics, but the models its theories work with are not like Fine's. In particular, their treatments of negation and conditionals in natural languages are incompatible with Fine's. They also take adverbs such as 'definitely', 'determinately', and 'clearly' seriously, rather than banishing them from the language. How can Fine's approach be reconciled with established formal semantic theories of natural languages?

A related challenge concerns reasoning in natural languages. As seen above, Fine's semantics invalidates various forms of argument which look good pre-theoretically, such as that from 'It's not both F and G' and 'It's F' to 'It's not G'. Why do such arguments look so good? One sort of response would be a roughshod error theory, but that is not Fine's usual style. He generally treats ordinary reactions with much more respect. Of the characterization of vagueness in terms of borderline cases, he writes: 'This position has become so entrenched that it is necessary to restore ourselves to a state of pre-theoretical innocence if we are to appreciate that there is a genuine issue as to whether it should be adopted' (p. 25). Against supervaluationists and epistemicists who take there to be a sharp cutoff in a sorites series, he presses very hard the question ‘But then why are we so inclined to think otherwise?’ (p. 17). In his discussion of the anti-luminosity argument, he makes much of how judgements involving numerous iterations of a knowledge operator look pre-theoretically (according to him), despite the difficulty of processing such iterations semantically (pp. 59-60).

However, when Fine comes to explain why we are tempted by illicit tolerance principles (such as ‘If *n* grains make a heap then *n*–1 grains make a heap), he posits ‘a transcendental illusion in something like the Kantian sense’, which ‘arises from thinking that we can attain an external or “transcendent” perspective on some phenomenon or practice from which no such perspective is to be had’ (p. 51). This illusion is also supposed to explain why (v) appears to define a meaningful operator. Well, such a transcendental illusion *might* explain why tolerance principles have tempted a few philosophers. But the main phenomena of vagueness are much more common and much less cerebral than that. No transcendental illusion is needed to make one feel that one is on a slippery slope. Elsewhere, I have shown how tolerance principles can arise from natural heuristics for vague terms: quick and easy cognitive methods, reliable enough for ordinary purposes but not perfectly reliable, which do not wear their fallibility on their face (Williamson 2020: 63-67). At any rate, what philosophers say about the psychology of vagueness should be plausible by the standards of cognitive psychology.

Fine’s book is far cleverer, more creative, better thought through, and more intellectually exciting than most of the literature on vagueness. That it is a step towards the truth about vagueness, I doubt.

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