1. Introduction

We are used to expressing and conceiving norms in deontic modal terms. They concern what one may or must (or should, or ought or has to) do or not do, be or not be. What one may do is permissible; what one must do is obligatory (not permissible not to do); what one must not do is impermissible or forbidden (not permissible to do). The semantic study of modal operators in natural language constitutes a mature research programme, whose explanatory power has been applied to understanding how such deontic modal terms work, and what logical principles they obey (Kratzer (1977, 1981, 2012), Portner (2009), Charlow and Chrisman (2016)).

But, this paper argues, there is a catch. Deontic modals are less perspicuous than they seem for articulating norms. The reason is that, as modal operators, they have a dimension of complexity extrinsic to the norms. In practice, this extra dimension has sometimes distracted theorists of norms, and sent them off on wild goose chases. We do better expressing and conceiving the norms in simpler, more perspicuous, non-modal terms. Although we can still recapture the corresponding deontic modal operators, that is a sideshow. The second half of the paper applies this framework to clarify some confusions which have sent the debate on norms of belief off on some unrewarding detours.

2. Compliance and permissibility

Here is a simple norm, a rule, expressed sub-sententially:

Rule R No dancing in the library.

To keep things simple, we consider normativity just with respect to rule R, ignoring all other rules or norms. For example, although making a rule such as R inevitably has moral consequences, we bracket them. R is more similar to a legal rule than to a moral one. In realistic cases, norms of several kinds and different priorities interact in various complex ways. Such interactions constitute an important topic in their own right, but for present purposes it is best to screen out such effects, in order to isolate an issue which does not depend on them.

A rule such as R induces a binary distinction between the permissible and the impermissible. Most obviously, it makes dancing in the library impermissible. For present
purposes, we can ignore more finely graded normative distinctions between better and worse.

2.1 Permissibility in context

Consider Alexandria, who is working in the library, not dancing. That is permissible. But is it permissible for Alexandria to dance? The natural answer is ‘No—she’s in the library’.

Here is another question, also asked while Alexandria is working in the library, not dancing: is it permissible for Alexandria to leave the library and dance? The natural answer is ‘Yes—the rule says nothing about what you can do outside the library’.

Now we have a puzzle, for we have rejected (1) and accepted (2):

(1) It is permissible for Alexandria to dance.

(2) It is permissible for Alexandria to leave the library and dance.

In other words, though Alexandria may leave the library and dance, she must not dance. But, given standard semantic treatments of the two sentences, (2) logically entails (1). More generally, standard semantic accounts validate these forms of argument:

P(&E) From ‘It is permissible for S to φ and ψ’, derive ‘It is permissible for S to φ’ and ‘It is permissible for S to ψ’.

We can put the reason like this, ignoring various subtleties in the semantics which do not matter here. The premise means that S φs and ψs in some relevant permitted situation s, where a situation is permitted just in case it contains no violation of R. Suppose that the premise is true. Then S φs in s and S ψs in s. So S φs in some relevant permitted situation and S ψs in some relevant permitted situation. But those are what the conclusions mean. Thus both conclusions are true. Therefore P(&E) is guaranteed truth-preserving.

The situation s at need not be actual. It may be merely possible, in a suitably broad sense of ‘possible’.

The treatment of ‘permissible’ as an existential quantifier is natural, and standard in deontic logic. After all, if it is impermissible for S to φ, then no relevant situation in which S φs is permitted. Conversely, if it is permissible for S to φ, then it is permissible for S to φ in some specific way, so the situation of S φing in that way is permitted. In terms of modal logic, P(&E) corresponds to the inference from ◊(α & β) to ◊α or ◊β, which is valid in any normal modal logic, and indeed in any monotonic modal logic (Chellas 1980).

Technically, one can devise non-monotonic deontic logics in which ◊(α & β) does not in general entail either ◊α or ◊β, and even provide them with a possible-worlds model theory of sorts in the framework of neighbourhood semantics (again, Chellas 1980). The reason for not going that way is explained below.

The motivation for P(&E) was put in terms of possible situations, rather than possible worlds, because it takes much less for a possible situation to avoid violations of the rule
than it does for a possible world to avoid them: worlds are global, situations are local. For example, imagine that in all the relevant worlds Jane obsessively dances in the library, so R is always violated. That should not affect the permissibility of Alexandria’s dancing. We can achieve this effect by taking account of possible small situations containing Alexandria’s actions but not Jane’s, so they need not contain violations of R. Possible world semantics can achieve the same effect, but less directly. Since most semantic accounts of modals use worlds rather than situations, we usually do the same in this paper, for ease of comparison, and achieve the ‘localizing’ effect of situations by explicit relativization to specific features of the situation, such as the agent.

Someone might object that the argument from (2) to (1) is not a genuine instance of P(&E), because it equivocates about the time of the dancing: (1) concerns what it is permissible for Alexandria to do now, while (2) concerns what it is permissible for her to do later. But time is not really the key to the problem. Both (1) and (2) are naturally understood as about the near future, not the present instant. When we ask ‘Is it permissible for Alexandria to dance?’, our interest is not always in whether it is permissible for her to be already dancing. For example, it may be common ground in the conversation that some dance music will start in two minutes, at noon, loud enough to be heard both inside and outside the library. Stopping work and leaving the library will take Alexandria less than two minutes. Thus the time of the dancing in both (1) and (2) may be naturally understood as starting from noon.

Nevertheless, such examples pose no threat to the validity of P(&E). For that means only that if the premise is true as uttered in a given context, the conclusions are true as uttered in the same context. When we originally considered (1), we held Alexandria’s location in the library fixed—it was not being questioned. When we went on to consider (2), we no longer held her location in the library fixed, for it was explicitly at issue. Once we have accepted (2), it is natural for us to change our answer to the original question ‘Is it permissible for Alexandria to dance?’, saying ‘Yes’, though perhaps adding by way of explanation ‘but first she must leave the library’. This fits the semantics of ‘permissible’ sketched above. Situations in which she left the library were implicitly excluded as irrelevant in the original context, but not in the later context. In any context in which (2) is true, (1) is also true. Standard semantic accounts of modals allow for just such context-sensitivity: the relevant possible worlds or situations are those in the contextually determined modal base, to use Kratzer’s terminology.

The reversal of attitude to (1) when (2) comes into play is a typical contextual effect. It is a special case of the more general conversational phenomenon by which it is easier to expand the domain of contextually relevant items than to contract it again. In this case, the items are possible situations or worlds. Once one has accepted (2), to continue plainly rejecting (1) suggests a conversational tin ear. A similar effect has been observed when extra conjuncts are added to the antecedent of a counterfactual conditional; the analogous pairs to (2) and (1) are known as reverse Sobel sequences, and the effect is evidence of context-sensitivity in the semantics of such conditionals (von Fintel 2001, Gillies 2007; for a response to Moss 2012’s attempt to explain away the effect in epistemic terms see Williamson 2020, 222-228). Hence (1)’s loss of acceptability on the introduction of (2) is explicable on pragmatic grounds, as a special case of an independently attested effect, with
no threat to the structurally attractive principle \( P(\&E) \). By contrast, using neighbourhood semantics to invalidate \( P(\&E) \) leaves the dynamic effect unexplained. Thus the evidence strongly favours a contextual account of the apparent gap between (1) and (2).

Contextual variation has been much debated in the recent literature on ‘ought’ and other deontic modals (Worsnip (2019) provides a helpful survey). However, the focus has not been on variation in the modal base. Instead, it has concerned what kind of normativity is intended—moral, legal, prudential, …—and whether the relevant standards vary with the values of participants in the conversation. In Kratzer’s terminology, the variation at issue is in the ordering source for ‘ought’. Variation in the modal base has attracted much less attention.

Context-sensitivity in the modal base is commonplace in natural language. But it has non-trivial methodological consequences for inquiry into norms. Consider a theorist who seeks a general necessary and sufficient condition for some sort of permissibility, perhaps for the toy example where only rule R is in play. If the candidate condition is formulated in contextually invariant terms, it will fall to counterexamples in at least some contexts, since ‘permissible’ is contextually variable: any expression cross-contextually equivalent to a contextually invariant expression is itself contextually invariant. On the other hand, if the candidate condition is formulated in contextually variable terms, then which condition is being discussed varies independently of the norm(s) under discussion. For the content of the theoretical formulation varies with the second-order context in which the theorists are discussing it, rather than with the first-order context in which those committed to the norm apply it. Of course, one can switch to a meta-linguistic formulation, which mentions the deontic modals rather than using them, and explicitly relativizes to a context of utterance. But that will not happen unless the need for it is recognized. Moreover, such metalinguistic formulations are often a complicating distraction from the meta-normative rather than meta-linguistic question originally at issue.

For example, rule R just says ‘No dancing in the library’: whether a given possible situation or world contains a violation of R does not vary with a contextually determined modal base. Although R does contain contextually variable elements, such as the reference of ‘the library’, they depend on the context where R is propounded, not on the context where ‘permissibility’ with respect to R is discussed—the latter is a different level of context-sensitivity.

2.2 Compliance conditions

We can display the non-modal nature of rule R by articulating a compliance condition for R. To avoid the complexities of situation semantics, while achieving a similar localizing effect, we work instead with triples of a subject \( S \), a time \( t \), and a world \( w \). The condition is simple:

\[
\text{COMPLIANCE } S \text{ complies with } R \text{ at } t \text{ in } w \text{ if and only if } S \text{ is not dancing in the library at } t \text{ in } w.
\]

We abbreviate ‘\( S \) complies with \( R \) at \( t \) in \( w \)’ as ‘Complies\(_{t,w}(S)\)’. To avoid clutter, the norm parameter ‘\( R \)’ is left implicit; it can be understood below as supplied by context.
Evidently, the compliance condition itself does not depend on a modal base. Of course, COMPLIANCE is intended to hold counterfactually, as well as actually, with the content of rule R being kept fixed, but the same holds for any other claim of necessity and sufficiency: it has nothing specific to do with the condition for which necessity and sufficiency are being claimed. Neither the left-hand side nor the right-hand side of COMPLIANCE expresses a modal condition in any distinctive sense. There is nothing especially modal about not dancing in the library. More precisely, the condition for compliance at a time t in a world w with respect to R, ‘S is not dancing in the library at t in w’, involves no quantification over worlds, whereas the conditions for permissibility and impermissibility at t in w with respect to R do involve quantification over worlds.

We can understand *violating* a rule as simply not complying with it. Thus:

\[
\text{VIOLATION } \quad S \text{ violates } R \text{ at } t \text{ in } w \text{ if and only if } S \text{ is dancing in the library at } t \text{ in } w.
\]

Obviously, violation is just as non-modal as compliance.

Obviously, some rules have a specifically modal content. An example is RM:

\[
\text{RM } \quad \text{Make as little noise as possible!}
\]

The condition for complying with RM is to make as little noise as possible; the condition for violating RM is to not make as little noise as possible. Those are both modal conditions, but that just concerns the specific content of RM, not its general status as a rule.

Of course, the mere statement that Alexandria complied with rule R is normatively neutral, in the sense that making it does not imply that R has any sort of normative authority. The same goes for the statement that she violated R. Once one accepts R’s authority, one needs some way to express it, and the normative positions it puts agents in. The salient worry is that we need deontic modals for that purpose, for example to say ‘She ought to comply with the rule’, so that falling back on compliance and violation avoids the complexities of deontic modals only at the cost of giving up normativity altogether.

In response, an initial point is that the normative nature of deontic modals is independent of their contextual variation with the modal base; the variation is neither necessary nor sufficient for the normativity. It is not sufficient, because non-deontic modals such as the ‘can’ of ability also vary contextually with the modal base. It is not necessary, because holding the modal base fixed—for example, to contain all metaphysically possible worlds—would not somehow neutralize the normative force of deontic modals. In principle, therefore, a term could do the normative work without the contextual variation. Indeed, we can judge someone’s past action or omission ‘right’ or ‘wrong’, ‘appropriate’ or ‘inappropriate’, ‘correct’ or ‘incorrect’, ‘OK’ or ‘not OK’, in virtue of its having complied with or violated a norm, without needing a modal base to make sense of the judgment. For instance, after Alexandria danced in the library, the librarian may judge her action ‘wrong’, ‘inappropriate’, ‘incorrect’, or ‘not OK’. Thus we can put a normative spin on compliance and violation without recourse to deontic modals.
2.3 Recovering permissibility from compliance

Given a normative standard of compliance, we can then combine it with a contextually specified modal base to recapture the corresponding deontic modality. Since compliance and violation have been characterized in effect over triples \(<S, t, w>\) of a subject, a time, and a world, we treat contextual relevance as a constraint on such triples. But instead of excluding some triples as absolutely irrelevant in the context, outside the domain of quantification (the analogue of the modal base), we treat contextual relevance as a contextually determined binary relation between triples. For example, consider (3), uttered in a context where it expresses a truth:

(3) It is not permissible for Alexandria to dance, but if she were outside the library it would be permissible for her to dance.

The first conjunct treats possible situations where Alexandria is outside the library as contextually irrelevant to permissibility in the actual situation, while the second conjunct treats them as contextually relevant to permissibility in a counterfactual situation where she is already outside the library. Similarly, consider (4), uttered in a context where it expresses a truth:

(4) It is not permissible for Alexandria to dance, but before she entered the library it was permissible for her to dance.

The first conjunct treats past situations when Alexandria was outside the library as contextually irrelevant to present permissibility, while the second conjunct treats them as contextually relevant to past permissibility before she entered the library. Since the relevance comes from sameness in Alexandria’s location, and in the conversation we may be contrasting what is permissible for her with what is permissible for other subjects, we also need the subject parameter \(S\) in the triples. However, we treat that parameter differently from the time and world parameters \(t\) and \(w\), holding the former fixed in the semantic clauses, when it is explicitly specified in the sentence, while letting the implicit parameters vary. A counterpart theorist such as David Lewis would let the explicit parameter vary too.

Beyond these bare structural constraints, we leave contextual relevance unanalyzed, since trying to give necessary and sufficient conditions for such messy pragmatic relations is a mug’s game, and in any case not to the present purpose.

Here is the result, with ‘\(\phi_{t^*,w^*(S)}\)’ abbreviating ‘\(S\) \(\phi\) at \(t^*\) in \(w^*\)’, and ‘contextually relevant’ occurring unanalyzed on the right-hand side:

**PERMISSIBILITY**

It is permissible with respect to \(R\) at \(t\) in \(w\) for \(S\) to \(\phi\) if and only if for some triple \(<S, t^*, w^*>\) contextually relevant to \(<S, t, w>\) such that \(\text{Complies}_{t^*, w^*}(S, \phi_{t^*, w^*(S)})\).
Here \( t^* \) and \( w^* \) are a time and world at which the putatively permitted action is performed, whereas \( t \) and \( w \) are a time and world at which it is putatively permissible: as already noted, when we ask whether something is permissible, we often have in mind doing it in the near future rather than already doing it, and many things are permissible without actually being done. We abbreviate the left-hand side as ‘Permissible\(_{t,w}(\phi(S))\)’.

In any context where relevance is a reflexive relation, PERMISSIBILITY makes compliance and performance together imply permissibility. More precisely, Permissible\(_{t,w}(\phi(S))\) whenever Complies\(_{t,w}(S)\) and \( \phi\)\(_{t,w}(S)\) because \( <S, t, w> \) is contextually relevant to itself.

It is also easy to check that PERMISSIBILITY validates the principle P(&E), for virtually the same reason as before.

We can define obligation as the dual of permissibility in the usual way:

\[
\text{OBLIGATORINESS } \quad \text{It is obligatory with respect to } R \text{ at } t \text{ in } w \text{ for } S \text{ to } \phi \text{ if and only if for every triple } <S, t^*, w^*> \text{ contextually relevant to } <S, t, w> \text{ such that Complies}\(_{t^*,w^*}(S)\) \text{ and } \phi\(_{t^*,w^*}(S)\). 
\]

We abbreviate the left-hand side of OBLIGATORINESS as ‘Obligatory\(_{t,w}(\phi(S))\)’. Thus, with \( \sim \) for negation, Obligatory\(_{t,w}(\phi(S))\) is equivalent to \( \sim\text{Permissible}\(_{t,w}(\sim\phi(S))\) \text{ and Permissible}\(_{t,w}(\phi(S))\) to \( \sim\text{Obligatory}\(_{t,w}(\sim\phi(S))\). 

As expected, in any context where relevance is reflexive, compliance and obligation together imply performance. More precisely, \( \phi\)\(_{t,w}(S)\) whenever Complies\(_{t,w}(S)\) and Obligatory\(_{t,w}(\phi(S))\).

In the usual way, we can make the time-world parameters implicit in a tensed modal language where formulas are evaluated at time-world pairs, and contextual relevance is treated like an accessibility relation between such pairs. However, in line with the earlier discussion, we keep an explicit parameter \( S \) for the subject, subscripted to the deontic modal operators, and prefix it to the world-time pairs to use the original relation of contextual relevance between subject-time-world triples. To emphasize the modal analogies, we use \( \Diamond S \) for ‘it is permissible for \( S \)’ and \( \Box S \) for ‘it is obligatory for \( S \)’, formalizing ‘It is permissible for \( S \) to \( \phi \)’ as \( \Diamond S \phi(S) \) and ‘It is obligatory for \( S \) to \( \phi \)’ as \( \Box S \phi(S) \).

The semantic clauses for the operators are then:

\[
\Diamond S \alpha \text{ is true at } <t, w> \text{ if and only if for some } t^* \text{ and } w^* \text{ such that } <S, t^*, w^*> \text{ is contextually relevant to } <S, t, w> \text{ and Complies}\(_{t^*,w^*}(S)\): \alpha \text{ is true at } <t^*, w^*>.
\]

\[
\Box S \alpha \text{ is true at } <t, w> \text{ if and only if for every } t^* \text{ and } w^* \text{ such that } <S, t^*, w^*> \text{ is contextually relevant to } <S, t, w> \text{ and Complies}\(_{t^*,w^*}(S)\): \alpha \text{ is true at } <t^*, w^*>.
\]

As one can easily check, once ‘Complies’ was introduced, the rest of the development was rather general. Although PERMISSIBILITY and OBLIGATORINESS include the phrase ‘with respect to \( R \)’, nothing in their structure depends on the very specific content of rule \( R \). Many other rules or pluralities of rules would do instead, with corresponding understandings of ‘Complies’.
A further complexity is that whether a rule such as R is in force is itself contingent and temporary: R had to be introduced. Universal morality does not forbid dancing in the library. Yet OBLIGATORINESS makes compliance obligatory (with respect to R) at all times in all worlds: if $\alpha$ is true at exactly the pairs $<t, w>$ for which Complies$_{s,w}(S)$, then trivially $\Box_S \alpha$ is true at any time-world pair whatsoever. In effect, the semantics has treated rule R as absolutely given. To take account of the contingent and temporary nature of such norms, we can consider a higher-order norm: to comply with any norm relevantly similar to R and currently (temporarily, contingently) in force. That is itself a higher-order compliance condition, and a similar semantic development can be carried through at this higher level. However, to simplify the exposition, that extra dimension of complexity will be ignored in what follows. It can easily be reintroduced.

2.4 How local is compliance?

Unsurprisingly, one can construct tricky cases for the semantic framework just sketched of rules whose violations are hard to localize. Here is an example on the temporal dimension. Imagine a lax train company with this rule:

RT Any journey made without a ticket must subsequently be paid for.

There is no deadline. Even after the traveller’s death, someone else can pay for the journey. Although RT can be violated, if a journey is never paid for, it is hard to say when the violation occurs—at every time after the journey has started in such a world? At worst, one might have to fall back on a coarser-grained version of the semantics which eliminates the temporal parameter from compliance.

Similarly, violations are sometimes hard to localize on the dimension of the subject S. An office may have this rule:

RS At least one member of staff must be present on the mezzanine level at any time.

When no member of staff is present on the mezzanine level, RS is violated, but there may be no particular member of staff who is violating it. Perhaps one should just say that the members of staff collectively are violating RS, and allow plural subjects in the semantics.

How far the distinction between compliance and violation can be localized depends on the content of the rule. That complicates the picture. Some flexibility is needed in applying the theoretical template; it cannot always be done mechanically. But that is not to say that it sometimes cannot be applied at all. The non-modal distinction between compliance and violation still takes priority over the modal distinction between permissibility and impermissibility.
2.5 The normativity of compliance

Another worry about RS and RT is that they are both stated using a modal operator, ‘must’, read deontically. For this point, the difference between ‘must’ and ‘obligatory’ does not matter; restating RT and RS in terms of ‘obligatory’ makes little difference. If the rules which lay down the distinction between compliance and violation to begin with are formulated in terms of deontic modals, how can the distinction take priority over those very deontic modals?

A short response is that we need the distinction between compliance and violation to make sense of the deontic modals. This is not simply an appeal to the definitions just proposed. Start with the unanalyzed deontic modals themselves, and put yourself in an extreme context focused just on living in the present and the actual, with no interest in cross-time or cross-world comparisons. In this case, contextual relevance between triples is simply identity. By the standards of this context, if what one is doing in a situation counts as permissible for one, in that situation one is complying with the relevant norms; if what one is doing in the situation does not count as permissible for one, one is violating some of those norms. Thus the distinction between compliance and violation can be recovered from deontic modals in a suitable context.

In less favourable contexts, the distinction may not be recoverable, because some situations are ignored as entirely irrelevant in the application of deontic modals, so nothing can be gathered from the application of the deontic modals as to whether those situations comply with the norms or violate them.

Once we have the distinction between compliance and violation, we can put a suitable normative spin on it, as noted above. For example, we can use ‘OK’ for the desired normative spin, without making it a deontic modal. Then we can informally paraphrase RT and RS:

RTa Journeys made without a ticket and never subsequently paid for are not OK.

RSa Situations when no member of staff is present on the mezzanine level are not OK.

Although RTa and RSa lack the pomposity characteristic of regulations, they do the job.

In any case, the role of deontic modals in stating rules is not to introduce context-dependence; it is to make explicit the rules’ intended normative force. The context-dependence simply comes along for the ride. Indeed, the semantics above for the obligation operator □S makes ‘□S (S complies)’ true under any contextual restriction for the standard of compliance at issue; the same applies to statements of rules in deontic modal form. The contextual variability of deontic modals can be compared to the ever-present possibility of implicit contextual restrictions on the domains of quantifiers in natural language. That does not stop us from using those quantifiers as a convenient way of stating universal generalizations for which only the explicit restrictions are intended, without exploiting their potential for being implicitly contextually restricted; if absolutely every F is G, then a fortiori every F in a contextually restricted domain is G. We may likewise use deontic modals as a
convenient way to make norms explicit, without exploiting their similar potential for implicit contextual restriction.

3. Application to norms of belief

The considerations in section 2 alert us to problematic assumptions in the current debate on norms of belief (e.g. Chan 2013, Matheson and Vitz 2014). Norms of belief in turn cast light on problems in localizing violations of norms.

Belief is subject to norms: one can believe something rightly or wrongly, correctly or incorrectly, reasonably or unreasonably, rationally or irrationally, with or without justification, and so on. Some of those distinctions focus more on the content of the belief, others more on how it was formed or is maintained. Some philosophers speak of doxastic or epistemic virtues or vices, but there is no need to moralize: basic normative distinctions amongst beliefs apply even to the beliefs of non-human animals and very young children, to which a moralizing approach is quite inappropriate. A norm for belief is more fruitfully treated as distinguishing between defective and non-defective beliefs, perhaps in a broadly functional sense (see Williamson forthcoming for discussion). Identifying a basic norm of belief thus becomes a way of understanding the nature of belief—which supports an approach applicable to all beliefs, not just to those of conscious, responsible agents.

3.1 Some salient norms of belief

One salient candidate norm of belief is the truth norm. We can put it in the imperative:

TNB Believe p only if p is true!

If we treat TNB as the basic norm of belief, we can extract a corresponding standard of non-defectiveness from it:

TND If S believes p, S’s belief in p is non-defective if and only if p is true.

A more demanding candidate norm of belief is the knowledge norm:

KNB Believe p only if you know p!

Since true belief does not entail knowledge, complying with TNB does not entail complying with KNB. But since knowledge does entail truth, complying with KNB does entail complying with TNB. However, treating KNB as the basic norm of belief is incompatible with treating TNB as its basic norm, and the corresponding standard of non-defectiveness is incompatible with TND (they cannot both be necessary and sufficient for non-defectiveness in the same sense):
KND  If S believes $p$, S’s belief in $p$ is non-defective if and only if S knows $p$.

Another less demanding candidate norm of belief is the evidence norm:

ENB  Believe $p$ only if you have good evidence for $p$!

‘Good evidence’ is so understood here that one can count as having good evidence for a falsehood. Thus complying with ENB does not entail complying with TNB, and so a fortiori does not entail complying with KNB. We also understand ‘good evidence’ so that one counts as having good evidence for $p$ whenever one knows $p$. Thus complying with KNB entails complying with ENB. However, treating ENB as the basic norm of belief is incompatible with treating either TNB or KNB as its basic norm, and the corresponding standard of non-defectiveness is incompatible both with TND and with KND:

END  If S believes $p$, S’s belief in $p$ is non-defective if and only if S has good evidence for $p$.

Of course, one can construct many other candidate norms of belief, even candidate basic norms of belief. This paper is not concerned with deciding between these candidates, but with the structure of the issues.

3.2 The permissibility of belief and Moore paradoxes

Some participants in the debate on norms of belief insist that a proper account of them must answer the question ‘When is it permissible to believe $p$?’ They want a necessary and sufficient condition for the permissibility of belief. Clearly, not even a necessary and sufficient condition for non-defectiveness such as TND, KND, or END specifies a necessary and sufficient condition for permissibility, since they apply only when S does believe. Even when S lacks belief in $p$, it may still be permissible for S to believe $p$, but a condition for non-defectiveness does not say when.

Of course, one can propose conditions for permissibility mechanically corresponding to conditions for non-defectiveness:

TNP  It is permissible for S to believe $p$ if and only if $p$ is true.

KNP  It is permissible for S to believe $p$ if and only if S knows $p$.

ENP  It is permissible for S to believe $p$ if and only if S has good evidence for $p$.

These are narrow-scope norms: the scope of ‘permissible’ extends only as far as ‘believe $p$’. But TNP, KNP, and ENP are no mere reformulations of the previous principles: they introduce special problems of their own.

A much-discussed example in the debate applies the truth norm TNP to Moore paradoxes (Bykvist and Hattiangadi 2007). Imagine that it is raining, but for some reason I
fail to believe that it is raining. Let \( p \) be the conjunction that it is raining and I fail to believe that it is raining. Thus \( p \) is true. Hence, by the right-to-left direction of TNP, it is permissible for me to believe \( p \), in other words, permissible for me to believe that it is raining and I fail to believe that it is raining. But whenever I believe the conjunction, I believe its first conjunct, so the second conjunct is false, so the whole conjunction is false. Thus it is permissible for me to believe \( p \) even though, whenever I believe \( p \), \( p \) is false. That is obviously quite at odds with the spirit of the truth norm of belief. In response, philosophers have proposed inserting various epicycles into TNP.

The difficulty comes from the modal nature of permissibility: the possible situation where it is permissible to believe \( p \) differs from the possible situation where \( p \) is believed. Recall Alexandria, sitting in the library. In the spirit of the truth norm of belief, is it permissible for her to believe that she is dancing? The natural answer is ‘No’, for the belief would be false. But, still in the spirit of the truth norm (and no longer worrying about rule R), is it permissible for Alexandria to start dancing and believe that she is dancing? The natural answer is ‘Yes’, for the belief would be true. We again have a puzzle, for we have rejected (5) and accepted (6):

(5) It is permissible for Alexandria to believe that she is dancing.
(6) It is permissible for Alexandria to start dancing and believe that she is dancing.

But (6) entails (5), just as (2) entails (1) in the original puzzle, by principle P(&E). For the same sort of reason as in section 2, there need be no equivocation about when she putatively believes that she is dancing: the example can be set up to make the time exactly the same for (5) and (6).

Just as before, the puzzle is an artefact of contextual variation in the modal base for the deontic modal ‘permissible’. In the original context when we assessed (5), we kept Alexandria’s bodily movements fixed, and treated possible situations where she is dancing as irrelevant. In the context when we assess (6), we no longer keep Alexandria’s bodily movements fixed, because they are explicitly at issue. We treat possible situations where she is dancing as relevant. We revise our attitude to (5) correspondingly. To the question ‘Is it permissible for Alexandria to believe that she is dancing?’, we might now answer ‘Yes’, adding by way of explanation ‘but first she must start dancing’.

Complicating TNP with epicycles to handle tricky propositions \( p \) is therefore a waste of time. For no such epicycles will handle the contextual variability of ‘permissible’, which depends not only on \( p \) but on many other factors too, including which other propositions happen to be salient in the context. As usual, attempts to handle pragmatic phenomena in the semantics end badly.

Naturally, the issue is not confined to the truth norm; it arises for other putative norms of belief too. KNP and ENP are just as vulnerable as TNP to contextual variability. Indeed, the same example works for any reasonable norm of belief. In the possible situations relevant for the original context, not only is it not true that Alexandria is dancing; she does not know that she is dancing, has no good evidence that she is dancing, and so on. By contrast, in some of the possible situations relevant for the later context, not only is it
true that Alexandria is dancing; she knows that she is dancing, has good evidence that she is dancing, and so on. Thus the contextual variation in the truth-value of (5) will occur on just about any reasonable norm of belief.

3.3 Compliant belief

We therefore have good reason to apply the framework of section 2 to norms of belief. As usual, the distinction between compliance and violation is primary. Norms such as TNB/TND, KNB/KND, and ENB/END are clear about what constitutes a violation: believing without truth, without knowledge, and without good evidence, respectively. That is what really matters. The rest is just a matter of finding the best way to fit deontic modals on top of that.

However, we must be careful how we do it. Let us try applying PERMISSIBILITY just as it stands in section 2. That requires cashing out ‘Complies_{t,w} (S)’ for a given norm of truth. What is it for a subject S to comply with such a norm at a time t in a world w? For the truth norm, the natural answer is this: for every proposition p such that S believes p at t in w, p is true. For the knowledge norm, it is: for every proposition p such that S believes p at t in w, S knows p at t in w. For the evidence norm it is: for every proposition p such that S believes p at t in w, S has good evidence for p at t in w. By plugging ‘the given norm of belief’ for ‘R’ and ‘believe φ’ for ‘φ’ into PERMISSIBILITY above, we then obtain this equivalence (for the given norm of belief):

(7) It is permissible at t in w for S to believe p if and only if for some triple <S, t*, w*> contextually relevant to <S, t, w> such that Complies_{t*, w*} (S), S believes p at t* in w*.

The development can subsequently proceed as in section 2.

However, that literal application of the template faces a problem. We first consider it for the truth norm. Let p be the proposition ‘Not everything I believe is true’. Suppose that I believe p. Then either p is not true, so not everything I believe is true, or p is true, so not everything I believe is true (for that is what p says). Either way, not everything I believe is true. Thus believing p guarantees that I violate the truth norm. In effect, ‘Complies_{t*, w*} (I)’ is incompatible with ‘I believe p at t* in w*’. Consequently, by (7), it is not permissible for me to believe p. But, whenever I believe p, since not everything I believe is true, p is true (because that is what it says). Thus my belief in p itself satisfies the truth norm, even though it guarantees that I will violate the truth norm elsewhere. There is a defect, but not in p. The condition for permissibility in (7) fails to capture that.

The problem is not confined to the truth norm. Since knowledge entails truth, the same argument from (7) shows that my belief in p is also not permissible with respect to the knowledge norm. Yet I may even know p, for example because I know that I have inconsistent beliefs. Thus my belief in p itself satisfies the knowledge norm too, even though it guarantees that I will violate the knowledge norm elsewhere. Again, there is a defect, but not in p. The condition for permissibility in (7) fails to capture that too. The evidence norm may also raise similar problems.
The argument makes a contentious assumption: that the sentence ‘Not everything I believe is true’ expresses the same proposition as used at different times \(t\) and \(t^*\). Arthur Prior would accept that assumption; many other philosophical logicians would not. If \(p\) is simply the proposition that not everything I believe at \(t\) is true, then the reasoning about its consequences for my beliefs at \(t^*\) does not go through. However, we can accommodate the alternative view of propositions by taking \(p^T\) to be instead the proposition that at each time in the interval \(T\) not everything I believe is true, where \(T\) includes every time \(t^*\) contextually relevant to \(t\) (with respect to some worlds). Then a variant of the reasoning still goes through. For suppose that I believe \(p^T\) at a relevant time \(t^*\) (in a world \(w^*\)). Hence either \(p^T\) is not true, in which case not everything I believe at \(t^*\) is true, or \(p^T\) is true, so at each time in \(T\) not everything I believe is true (for that is what \(p^T\) says), so not everything I believe at \(t^*\) is true (for \(t^*\) is in \(T\)). Either way, not everything I believe at \(t^*\) is true. Thus believing \(p^T\) guarantees that I violate the truth norm at \(t^*\). In effect, ‘Complies\(_{t^*,w^*}(I)\’ is incompatible with ‘I believe \(p^T\) at \(t^*\) in \(w^*\).’ Consequently, by (7), it is not permissible for me to believe \(p^T\) at \(t\). Yet \(p^T\) may well be true; indeed, it will be true if I believe it throughout the interval \(T\). I may even know \(p^T\), through my awareness of the deep-seated inconsistencies in my belief system (we all have them). We can choose a context where the interval \(T\) is short enough for such knowledge to be available, and raise the problem with respect to that context. The use of Moore’s paradox as a counterexample to TNP may require similar tweaking.

A structurally similar issue arises for the norm of promise-keeping. I promise you that I will break a promise. If I keep that promise to you, I break another promise. Just by making that promise, I am guaranteed to violate the norm of promise-keeping. Nevertheless, I can keep the promise to you. For the norm of promising, we track the distinction between compliance and violation promise by promise, not just promiser by promiser. Similarly, for norms of believing, we track the distinction between compliance and violation belief by belief, not just believer by believer.

One way to handle the issue is by localizing compliance and violation to a proposition as well as to a subject, a time and a world. Thus we may write ‘Complies\(_{t,w}(S, p)\’ in place of ‘Complies\(_{t,w}(S)\’. For the truth norm, the compliance condition with respect to a quadruple \(<S, p, t, w>\) is: if S believes \(p\) at \(t\) in \(w\), \(p\) is true at \(t\) in \(w\) (with ‘if’ read materially). For the knowledge norm, it is: if S believes \(p\) at \(t\) in \(w\), S knows \(p\) at \(t\) in \(w\). For the evidence norm, it is: if S believes \(p\) at \(t\) in \(w\), S has good evidence for \(p\) at \(t\) in \(w\). And so on. We also treat the explicitly specified proposition \(p\) in the same way as the explicitly specified subject \(S\) with respect to the contextual relevance relation. Thus in place of (7) we have (8) (for the given norm of belief):

\[
(8) \quad \text{It is permissible at } t \text{ in } w \text{ for } S \text{ to believe } p \text{ if and only if for some quadruple } <S, p, t^*, w^*> \text{ contextually relevant to } <S, p, t, w> \text{ such that Complies}_{t^*,w^*}(S, p), S \text{ believes } p \text{ at } t^* \text{ in } w^*.
\]

This handles the permissibility of beliefs like ‘Not everything I believe is true’ in the desired way.

More generally, within this framework we can define deontic modal operators \(\Diamond_{S,p}\) and \(\Box_{S,p}\) relativized to a subject \(S\) and proposition \(p\):
◊_{S, p} \alpha \text{ is true at } <t, w> \text{ if and only if for some } t^* \text{ and } w^* \text{ such that } <S, p, t^*, w^*> \text{ is contextually relevant to } <S, p, t, w> \text{ and Complies}_{t^*, w^*}(S, p): \alpha \text{ is true at } <t^*, w^*>.

□_{S, p} \alpha \text{ is true at } <t, w> \text{ if and only if for every } t^* \text{ and } w^* \text{ such that } <S, p, t^*, w^*> \text{ is contextually relevant to } <S, p, t, w> \text{ and Complies}_{t^*, w^*}(S, p): \alpha \text{ is true at } <t^*, w^*>.

In particular, we understand ‘It is permissible for S to believe } p\text{’ (or ‘S may believe } p\text{’, read deontically) as ‘◊_{S, p} (S believes } p\text{)’. Likewise, we understand ‘It is obligatory for S to believe } p\text{’ (or ‘S ought to believe } p\text{’) as ‘□_{S, p} (S believes } p\text{)’.

3.4 Obligations to believe?

Some explanation is needed of the truth-condition for ‘S ought to believe } p\text{’. As we have seen, the compliance condition for } S \text{ and } p \text{ will be of this form, for some } C: \text{ either } S \text{ fails to believe } p \text{ or } C(S, p). \text{ The truth-condition for ‘S ought to believe } p\text{’ is then that on every contextually relevant quadruple such that either } S \text{ fails to believe } p \text{ or } C(S, p), S \text{ believes } p. \text{ That condition fails whenever, on some relevant quadruple, } S \text{ fails to believe } p\text{; it holds whenever on every relevant quadruple } S \text{ believes } p. \text{ It therefore simplifies to the condition that for every contextually relevant quadruple, } S \text{ believes } p. \text{ The result is independent of the specific content of the compliance condition } C(S, p). \text{ All it depends on is the structural point that one vacuously complies by failing to believe, for the only case where a norm of the kind at issue obliges one to believe is the trivial one where failure to believe is contextually irrelevant. It is like the well-known effect of the standard deontic modal semantics that it makes all tautologies trivially obligatory: a surprising result, but the surprise is easily explicable on pragmatic grounds: there is normally no conversational point in insisting that your interlocutor } ought \text{ to be such that } 2 + 2 = 4.

The non-trivial obligations generated by such norms of belief are only to satisfy the compliance condition at issue:

TNO  S ought to (believe } p\text{ only if } p \text{ is true)

KNO  S ought to (believes } p\text{ only if } S \text{ knows } p\text{)

ENO  S ought to (believe } p\text{ only if } S \text{ has good evidence for } p\text{)

These are wide-scope obligations. ‘S ought to’ is used informally for the obligation operator □_{S, p}. Given the corresponding compliance conditions, TNO, KNO, and ENO hold under any contextual restriction.

Of course, we can also consider putative norms on which belief is sometimes non-trivially obligatory. These too will be derivative from a corresponding compliance condition. For example, one such norm is violated by any unbelieved truth. That norm implies a wide-scope converse obligation to TNO (under any contextual restriction):
TNO\textsubscript{C} \quad \text{S ought to (believe } p \text{ if } p \text{ is true).}

As we should expect, TNO\textsubscript{C} does not imply the corresponding narrow-scope principle:

TNO\textsubscript{CN} \quad (S ought to believe } p \text{) if } p \text{ is true.}

One counterexample is a true Moore-paradoxical sentence in place of ‘}p\text{’. For since S’s believing } p \text{ falsifies } p \text{, S is not obliged to believe } p \text{; thus TNO\textsubscript{CN} has a false consequent and true antecedent. But the example poses no threat to TNO\textsubscript{C}, for S can meet that obligation simply by falsifying } p \text{, perhaps by believing that it is raining, contrary to the second conjunct of } p \text{.}

A more demanding norm is violated by any \textit{unknown} truth. Such a norm implies a wide-scope obligation connecting truth and knowledge (under any contextual restriction):

KO \quad S ought to (know } p \text{ if } p \text{ is true).

Predictably, KO does not imply the corresponding narrow-scope principle:

KO\textsubscript{N} \quad (S ought to know } p \text{) if } p \text{ is true.}

A true Moore-paradoxical sentence is also a counterexample to KO but not to KO\textsubscript{N}, for reasons similar to those for TNO\textsubscript{C} and TNO\textsubscript{CN}. A notable feature of KO is that the putative obligation does not specifically concern \textit{belief}; it simply relates truth and knowledge.

3.5 Fine-grained belief states

Another direction of inquiry is to experiment with making the framework even more fine-grained than it already is. For we can make some sense of the idea that a given subject at a given time has \textit{several} beliefs in the same proposition. For example, on a directly referential view of proper names, the sentences ‘Hesperus is bright’ and ‘Phosphorus is bright’ may express the same proposition. Someone who doubts the identity sentence ‘Hesperus = Phosphorus’ may believe a single proposition twice over, once under the mode of presentation ‘Hesperus is bright’ and once under the mode of presentation ‘Phosphorus is bright’. We could regard him as having two beliefs, psychologically individuated, in that proposition. That would make no difference to the truth norm, since the two beliefs would have the same truth-value. But it \textit{would} make a difference to many other putative norms of belief. For example, someone who accepts ‘Hesperus = Phosphorus’ on inadequate evidence may have two beliefs in the identity proposition: perhaps her belief in it under the mode of presentation ‘Hesperus = Hesperus’ constitutes knowledge, while her belief in it under the mode of presentation ‘Hesperus = Phosphorus’ does not. One belief complies with the knowledge and evidence norms while the other violates them, even though they are beliefs in the same proposition. Psychological compartmentalization, a ‘divided mind’, may produce
similar results even without a difference in mode of presentation. We can adapt the present approach to beliefs so individuated by reinterpreting the same formal framework, by understanding the variable ‘p’ as taking beliefs rather than propositions as values and tweaking the interpretation of ‘S believes’ accordingly. Many variations can be played on the same theme.

4. Conclusion

How far the distinction between compliance and violation can be localized depends on the specific nature of the norm at issue. For present purposes, we can leave that matter schematic, rather than seeking explicit uniformity. The moral to take away is that, when our interest is in the normative questions rather than the semantic ones, we should redirect our focus away from the modal distinction between permissibility and impermissibility, towards the non-modal distinction between compliance and violation.

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