Bolzano’s argument for the Identity of Indiscernibles

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“One of the most difficult questions is whether there must be some difference between any two objects (where there actually are two, and not just one object represented twice). One might already respond in the affirmative based upon the reason that for any two objects ALPHA and BETA there must at least obtain the difference that ALPHA has the attribute of "being ALPHA and not BETA'', and BETA the opposite attribute. But this is only an external difference. But it seems that if we ask whether there is a difference between any two objects, we would not be satisfied with a merely external difference between them (which would already obtain if we distinguished them in thought), but rather we ask whether there is an inner attribute which the one has and the other lacks. I am quite inclined to think that such an inner difference must hold between any two objects that have actual existence. In other words, I think that there are no two actual objects that equal one another in all their internal attributes [Bolzano added the following note here: “The reader will notice that I have referred to this theorem already several times under the name of the LEIBNIZIAN maxim de identitate indiscernibilium”] My reasons are as follows: Every finite substance is influenced by every other substance, no matter how far they are apart. Thus, even if we make the most extreme assumption, namely, that the original attributes of two substances, i.e., the attributes that stem from the Creator Himself, are completely equal, an undisturbed equality of the two substances can continue only so long as both are surrounded by equal substances acting upon them from the same distances, or if there is an inequality in one of these parts, then this difference must be of such a sort that its effects are the same for both substances. But both of these assumptions are infinitely improbable, because of the infinitely many different cases that have the same probability. It is certain that at least in most cases there is not only one, but many, nay infinitely many, external and internal differences that can be truly stated of two objects. The more closely we compare two people, or two trees, or any other two actual objects, the more differences we discover between them.”

1. Here Bolzano addresses the issue of the Identity of Indiscernibles and gives an argument for it. I agree that this is one of the most difficult questions in Philosophy. It is so difficult that it is easy to get confused even over the formulation of the issue. Indeed Bolzano is confused in more than one way.
2. First, the question of the Identity of Indiscernibles is not whether there must be some difference between any two objects, but whether there must be some extra-numerical difference between any two objects. Otherwise the answer is trivial: of course there must be some difference between any two objects, since if they are two objects they must differ at least numerically. This is why the Identity of Indiscernibles is canonically formulated, by Leibniz, as the thesis that no two objects can differ *solo numero*, i.e. no two objects can differ only numerically. In other words, every two objects must differ extra-numerically.
3. Bolzano is aware that there is something unsatisfactory with asserting the Identity of Indiscernibles on the basis that any two objects, ALPHA and BETA, will differ from each other in that one of them has the attribute of *being ALPHA and not BETA* and the other has the attribute of *being BETA and not ALPHA*.
4. But Bolzano’s dissatisfaction with this is that the said difference would be external. Presumably, the attribute of *being ALPHA and not BETA* is external because having it depends on a relation, the relation of non-identity, with respect to BETA. But Bolzano thinks that a difference between two objects is an attribute that is had by one of them but lacked by the other. So, there is no need to appeal to conjunctive attributes like *being ALPHA and not BETA*. One can support the Identity of Indiscernibles in basically the same way by appealing to attributes like *being ALPHA*. Since ALPHA, but not BETA, has this attribute, this attribute is a difference between ALPHA and BETA. Furthermore, it is an internal attribute, since ALPHA has the attribute of being ALPHA in and of itself (Bolzano, page 381). Furthermore, for Bolzano an external attribute is one such that having it consists in a relation to *another* object (Bolzano, pages 274-75) and having the attribute of *being ALPHA* does not consist in a relation to *another* object.
5. But why should we be dissatisfied with merely an external difference between two objects? Bolzano gives no reason. And there is indeed no reason for such a dissatisfaction. The principle that any two objects must differ extra-numerically, whether internally or externally, is a perfectly interesting principle, and not a trivial one.
6. Bolzano assimilates his principle to Leibniz’s principle. But although Leibniz would have agreed with the text of a principle that rules out objects that share all their internal attributes, he meant a different thing than Bolzano did. Indeed, Leibniz’s principle is stronger than Bolzano, since Leibniz thought that quantitative differences like size were not among the intrinsic properties which he thought must distinguish any two objects (See Rodriguez-Pereyra 2014: 29-37), while it seems that for Bolzano size is an inner or internal attribute (Bolzano page 277 says that the relation between the lengths of the sides of a triangle is an internal attribute of the triangle, but what is the size of a triangle if not that relation between the sides).
7. There is a second difference between Bolzano’s and Leibniz’s versions of the principle of Identity of Indiscernibles. This is that Bolzano is arguing only for the truth of the principle, while Leibniz asserted and argued for the *necessary* truth of the principle. The evidence that he was arguing for the truth rather than the necessity of the principle is not that he asserts that the principle must hold between any two actual objects. For by ‘actual object’ Bolzano means something like ‘concrete object’, rather than what contemporary philosophers mean by ‘actual object’. The evidence is that there is no modal qualifier in his statement of the Identity of Indiscernibles and that his argument is explicitly formulated to support the high probability of the Identity of Indiscernibles. (It might be thought the word “must” is the relevant modal qualifier; but it isn’t – what he means by using this word is that his argument is conclusive, but claiming that a certain argument is conclusive is not the same as claiming that the conclusion is necessary).
8. Bolzano’s argument can be reconstructed as follows:
9. The internal attributes of any finite substance are a causal effect of all other substances.
10. Therefore, for there to be no internal difference between two given substances they must not only have been given equal original attributes by God, but they must be surrounded by perfectly similar substances at the same distance, or by dissimilar substances that have exactly the same effects upon them.
11. For every distribution of substances across space in which two substances A and B are surrounded by perfectly similar substances at the same distance or in which A and B are surrounded by dissimilar substances causally affecting them in exactly the same manner, there are infinitely many other equiprobable distributions of substances across space in which A and B are surrounded by some dissimilar substances causally affecting them in a different way.
12. Therefore, it is infinitely improbable that any two substances A and B are surrounded by perfectly similar substances at the same distance or that they are surrounded by dissimilar substances that have exactly the same effect upon them.
13. Therefore, there are no two actual (i.e. concrete) objects that share all their qualitative internal attributes.
14. This is not a Leibnizian argument. For, in his mature philosophy, Leibniz rejected causal relations between finite substances. It is true that, although Leibniz rejected causation between finite substances, he maintained that all substances express each other, and that what happens in one mirrors what happens in every other. But as far as I am aware Leibniz never argued for the Identity of Indiscernibles from this principle. There is, however, an early text from 1676, *The Meditation on the Principle of the Individual*, where Leibniz makes some connections between the Identity of Indiscernibles and causation, but the connection is not with the idea that all substances causally affect each other but with the idea that the effect expresses its cause.
15. Bolzano is assuming that similar causes have similar effects and also that similar effects need not have similar causes. But this last assumption is in contradiction with something else Bolzano said in a different part of the Wissenschaftslehre, namely: “For it is quite in order that different causes should have different effects” (page 237). This is the contrapositive of, and therefore equivalent to, the claim that similar effects have similar causes. But then the attributes of perfectly similar substances cannot be the effects of dissimilar substances.
16. Third, there are at least two reasons why the argument, as formulated, is invalid. The first is that the conclusion is about all actual objects but the preceding premises support only a conclusion about all substances and Bolzano believed in actual objects other than substances, namely what he called “adherences”. Following Benjamin Schnieder (Roski and Schnieder 2022: 425), adherences are what contemporary philosophers would call tropes or modes, that is, features or attributes that inhere in other objects and that are dependent on them and are therefore unique to them. Thus adherences are unshared if not unshareable. It should be noted that there are limits to the assimilation of Bolzano’s adherences to tropes or modes, since Bolzano conceived of events as adherences too (Roski and Schnieder 2022: 14). In any case, I think it is possible to extend Bolzano’s reasoning to apply to adherences too. For what is common to all actual objects, that is, to substances and adherences, is that they cause effects and, if they are finite, that they suffer changes (para 79, page 366) and therefore have causes. Then Bolzano can apply the same reasons he gave for there being no perfectly similar substances to the case of adherences, namely that perfectly similar adherences must be surrounded by perfectly similar substances (i.e. substances having perfectly similar adherences) at the same distance, or by dissimilar substances (i.e. substances having dissimilar adherences) that have exactly the same effects upon them, etc. So, this invalidity of Bolzano’s argument is easily repaired.
17. But there is another reason why Bolzano’s argument is invalid. Let us start by noting that it is not clear how to make sense of Bolzano’s notion of “infinite improbability”: according to the standard axioms of probability, the lowest probability, and therefore the highest improbability, is 0. Consider, for instance, the classic example of a dart hitting a single point out of infinitely many. Such an event is normally taken to have probability 0, and yet it occurs. And it is very likely that Bolzano would have considered the event of a dart hitting a single point out of infinitely many as an infinitely improbable event. In any case, no matter how Bolzano thought of infinite improbability, the fact is that the most that follows from an event or fact being uttermost improbable is that it is… uttermost improbable – but not that it does not occur. So whatever degree of probability corresponds to Bolzano’s notion of infinite improbability, that an event has infinite improbability does not entail that it does not occur.
18. But perhaps Bolzano did not want to assert the truth of the Identity of Indiscernibles but only its high probability – that is, perhaps he meant to argue that the Identity of Indiscernibles is more probable than its negation. Some evidence for this might be found in the fact that Bolzano says that he is “quite inclined” to think that an inner difference must hold between any two objects that have actual existence. So let us suppose that what Bolzano was arguing was that it is highly probable that there are no two actual objects that share all their qualitative internal attributes. In that case the final premise must be reformulated thus:

5\* Therefore, it is more probable that there are no two actual (i.e. concrete) objects that share all their qualitative internal attributes than not.

1. There is a problem with premise 3. What it says is that, for every situation S in which there are two substances A and B surrounded by substances affecting them in the same way, (a) there are infinitely many other situations in which A and B are surrounded by substances affecting them in different ways, and (b) S and the infinitely many other situations are equiprobable.
2. But (b) is unsupported since it is based on an unsupported appeal to the Principle of Indifference, and there is no good argument that reality is indifferent with respect to the situations envisaged in premise 3. At most one can say that all such situations have the same priors, but what priors we assign to situations is a factor of our epistemic state: what we know about reality and what we find most plausible about it given what we know about it. And we do not know enough about reality to assign the same priors to such situations. The equiprobability claim, and therefore the third premise, will therefore remain unsupported if challenged by a denier of the Identity of Indiscernibles.
3. But even if we grant premise 3, and therefore grant (b), 5\* does not follow. For 3 says that situations in which there are indiscernible substances and situations in which there aren’t are equiprobable. Therefore it is not the case that situations in which there aren’t indiscernible substances are more probable than situations in which there aren’t. That is, it does not follow that the Identity of Indiscernibles is more probable than its negation. So Bolzano’s argument is still invalid even if we suppose that he was arguing only it is highly probable that there are no two actual objects that share all their qualitative internal attributes.
4. But there is another interpretation of premise 3. One might think that what Bolzano had in mind was the following:

3\*. There are infinitely many more situations in which two substances A and B are surrounded by substances affecting them in different ways than situations in which they are surrounded by substances affecting them in the same way, and all these situations are equiprobable.

1. But 3\* is false. For to every situation in which every situation in which any substances A and B are surrounded by dissimilar substances causally affecting them in a different way there corresponds a situation in which there are perfectly similar substances. For any distribution of substances across space is half of a qualitatively symmetric situation that has two indiscernible halves. Thus, take the actual distribution of substances across space and assume that there are no perfectly similar substances in it. Even so, there is a possible perfectly qualitatively symmetrical situation containing the actual distribution as one if its halves and so in that situation every substance is one of a pair of perfectly similar substances. And this is not true just of the actual distribution of substances across space but of every possible one.

1. But then the number of situations in which there are perfectly similar substances is not less than the number of situations in which any two substances A and B are surrounded by dissimilar substances causally affecting them in a different way. And so it is not the case that there are infinitely more situations in which any two substances A and B are surrounded by dissimilar substances causally affecting them in a different way than situations in which there are perfectly similar substances. Thus the conclusion 5\* does not follow.
2. In conclusion, Bolzano’s argument is invalid as an argument for the truth of the Identity of Indiscernibles and invalid or unsound as an argument for its high probability.